DOCUMENT RESUME

ED 064 052 SE 012 729

TITLE Mathematics Curriculum Guide, Sections H and I.
INSTITUTION Victoria Education Dept. (Australia).
PUB DATE 69
NOTE 144p.

EDRS PRICE MF-$0.65 HC-$6.58
DESCRIPTORS Arithmetic; *Curriculum; Curriculum Guides; Decimal Fractions; *Elementary School Mathematics; Fractions; Geometric Concepts; Graphs; Statistics
IDENTIFIERS Australia

ABSTRACT

This guide is the seventh of a series of seven curriculum guides included in SE 012 723, SE 012 724, and SE 012 725, and concludes the primary school mathematics course (grades one through six in Victorian schools, Australia). It is expected that "a substantial majority of children leaving grade six will have completed Section H. Some children will have had experience with Section I, while others will not have reached Section H." Perhaps for this reason, the majority of the activities suggested for these two sections are designed to produce deeper understanding of the previous material through extensions and further applications. New topics include rounding off, multiplication of fraction by fraction and of decimal by decimal, ratio and percentage, rotational symmetry, and histograms and averages (mean, mode and median). (MM)

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CURRICULUM GUIDE
MATHEMATICS

SECTIONS H and I

EDUCATION DEPARTMENT OF VICTORIA

CURRICULUM GUIDE

MATHEMATICS

SECTIONS H AND I

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INTRODUCTION

Sections H and I are the final parts of the primary schools mathematics course. They build upon the understandings, the skills, and the knowledge that have been acquired in the previous years. They also look forward to the mathematical experiences that will be acquired in the secondary school. The primary schools mathematics course provides a wide variety of specific experiences. These will provide the foundation on which later ideas of a general abstract nature can be developed. Secondary school algebra is prepared for through understandings gained in arithmetic. Formal geometry will become meaningful as a result of the child's previous familiarity with shapes and their simple attributes. Set language and theory in Form I can develop from the child's experiences involving classification and categorisation of the objects in his environment in preceding years.

Likewise, the search for pattern and structure that began in Section A carries through Sections H and I to a more generalised form in the secondary school. A growing awareness of pattern and structure at this level comes from matters such as an extension of the application of basic properties to fractions and decimals, and the extension of the study of notation to bases other than ten. In spatial relations this continuing study of pattern carries over into the symmetry properties of figures in Sections H and I.

It is expected that, in normal circumstances, a substantial majority of children leaving Grade VI will have completed Section H. Some children will have had experience with Section I, while others will not have reached Section H. The mathematics course is a sequential one, designed to enable each child to progress at his own rate so far as he is able ; no prescription concerning expected grade standards is possible or desirable.

Mathematical experiences can arise from a variety of situations. The environment can supply many interesting problems as this Guide will suggest. Another starting-point could be a current happening or a planned and structured experience with material such as a geoboard, Cuisenaire material, a spike abacus, or graph paper. The number line can be a valuable semi-abstract aid both for present purposes and for future use.

The traditional class, smaller groups, and individual activity all have something to contribute to the child's learning experiences. Many of the activities suggested in the Guide will have greatest value if carried out by two or more children working together.

This Guide, as with the earlier Guides, should in *no* way be regarded as being prescriptive. It suggests ways in which the course of study might be interpreted and implemented by the teacher in the classroom.

Current trends in curriculum development indicate the desirability of planning the child's educational experiences as a whole. Although mathematics has a unique contribution to make, much of it relates to other areas of knowledge and experience. Some of these common areas concern science, social studies, physical education, and language. For instance, physical education can provide data for measuring and recording ; science will provide numerous occasions for measurement activities and for experience in classification ; mathematics is expressed in a language that is unique, universal, and free from doubt and ambiguity.

Both the Course of Study and the Guide to it are organised in topics. This is done so that the sequence of development might be readily followed by teachers. However, it is expected that many will reorganise the material. Only the classroom teacher can decide on the organisation of experience best suited to the children of his class.

PATTERN AND ORDER IN THE NUMBER SYSTEM

SECTION H

In Section H, and subsequently in Section I, the counting activities set out in the Guide to Section G should be continued. The need to select specific ranges, limited to a minimum of numbers needed to establish or revise a particular skill or understanding, is again stressed.

Examples :

(i) Count from 993 to 1,014 by ones.
(ii) Count from 9,953 to 11,253 by hundreds.
(iii) Count from 2,327 to 2,507 by tens.
(iv) Count from 23·27 to 25·07 by tenths.

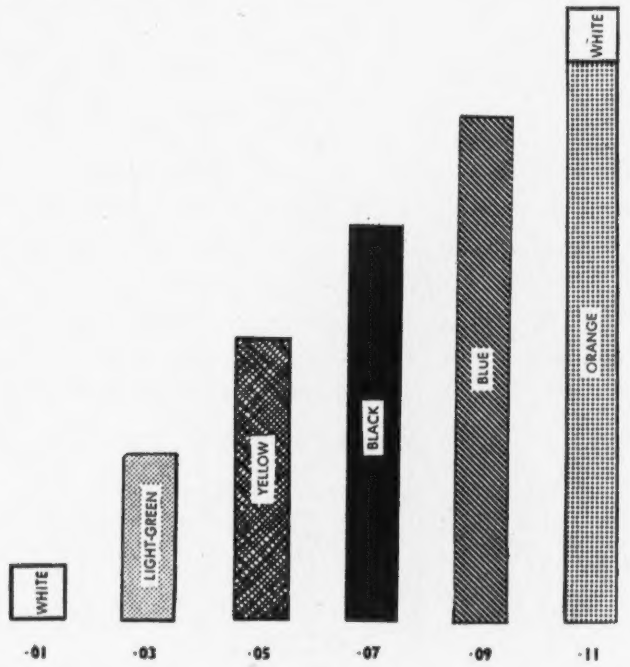
Selected counting ranges can be used to remedy deficiencies in a child's knowledge of number facts. It is necessary to maintain and increase the child's efficiency in correct response to number facts. Counting activities such as the following will be found useful :

(i) Count by eights from 7 to 63.
(ii) Count by nines from 117 to 63.

The abacus, the bead-frame, the "staircase" of rods, the number line, and counting charts or number boards can all be used to advantage in counting situations.

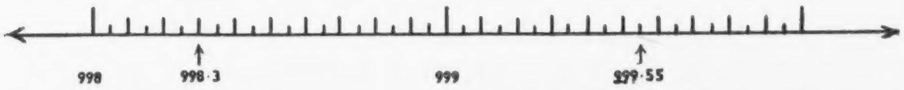
Example (i)

If the measure of orange is ·1, count up and down the "staircase" illustrated below :



Example (ii)

Count from 998·3 to 999·55 by the intervals marked on the number line below :



Ideas for investigation are suggested in the Guide to Section G (page 8) under the heading "Further Study of Patterns in Number Sequences". These ideas may well be pursued further. Patterns may now involve new understandings gained by the child in decimal fractions and in vulgar fractions. For example :

In the following examples replace the * by suitable numerals :

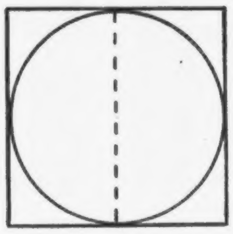
(i) ·9 ; 9 ; 90 ; * ; * ; * ; 900,000.
(ii) 750 ; * ; 7·5 ; * ; ·075.
(iii) 16 ; 3·2 ; ·64 ; *.
(iv) $2\frac{2}{3}$; $\frac{2}{3}$; * ; $\frac{1}{24}$; *.
(v) ·25 ; ·5 ; ·75 ; * ; * ; 1·5.
(vi) 5 ; 4·2 ; 3·4 ; * ; 1·8 ; *.
(vii) $1\frac{2}{3}$; $3\frac{1}{3}$; 5 ; $6\frac{2}{3}$; * ; 10 ; *.

Patterns can be discovered in tables of measures resulting from children's activities. For example, following the measurement of circumference and diameter of various circular shapes, the following table may result :

Circumference	Diameter
22 in.	7 in.
11 in.	$3\frac{3}{4}$ in.
$37\frac{3}{4}$ in.	12 in.
494 steps	159 steps

From inspection in each case it is seen that the circumference is about three times the length of the diameter.

In Section I a similar activity may involve the areas of circular and square shapes ; for example, when the length of a diameter is equal to that of a side of the square (see diagram below).



Further discussion of this activity occurs on page 84 of this Guide. Other instances where tables of related numbers occur are referred to in the Curriculum Guide to the primary science course (see *C. Branching Out*, page 98). Children should be given encouragement to discover for themselves relationships which exist between pairs of numbers.

Patterns seen in arrays of numerals have been discussed in the Guide to Section G (page 10). Activities such as these should be continued and extended in complexity to provide adequately for individual children. For example :

$\frac{1}{3}$	$1\frac{2}{3}$	$8\frac{1}{3}$	*	*	*	(× 5)
$\frac{1}{2}$	$1\frac{5}{6}$	*	$41\frac{5}{6}$	*	*	($\frac{1}{6}$ added to above number)
*	11	51	251	1251	*	(Number above × 6)
·6	2·2	10·2	*	250·2	*	(Number above ÷ 5)

Magic squares can be challenging and interesting exercises. A magic square is a square array of numbers such that all horizontal, vertical, and diagonal totals are equal.

For example :

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

In the magic square above, each row, each column, and each of the two main diagonals add to 65. This *basic* pattern is seen also in :

8	1	6
3	5	7
4	9	2

The amount of detail given to activities involving magic squares in this Guide should not suggest to the teacher that undue importance should be given to this topic. Detail is provided simply because many teachers may be unfamiliar with the exercise.

Various operations can be carried out on the magic square without destroying its magic properties. For example, take the basic square A :

A

8	1	6
3	5	7
4	9	2

Add 15 to each cell of A

23	16	21
18	20	22
19	24	17

Subtract 1 from each cell of A

7	0	5
2	4	6
3	8	1

Multiply each cell of A by 3

24	3	18
9	15	21
12	27	6

Divide each cell of A by 5

$1\frac{3}{5}$	$\frac{1}{5}$	$1\frac{1}{5}$
$\frac{3}{5}$	1	$1\frac{2}{5}$
$\frac{4}{5}$	$1\frac{4}{5}$	$\frac{2}{5}$

A magic square can be rotated through 1, 2, or 3 quarter turns, or reflected about a central row, a central column, or a diagonal, without altering its addition property. For example :

A

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

B

11	10	4	23	17
18	12	6	5	24
25	19	13	7	1
2	21	20	14	8
9	3	22	16	15

C

15	8	1	24	17
16	14	7	5	23
22	20	13	6	4
3	21	19	12	10
9	2	25	18	11

D

9	2	25	18	11
3	21	19	12	10
22	20	13	6	4
16	14	7	5	23
15	8	1	24	17

A is rotated through a quarter turn to obtain B.
B is reflected about a diagonal to obtain C.
C is reflected about a central axis to obtain D.

Enrichment

Some children will desire to probe further into number. These may well investigate Roman and/or Chinese systems of numeration.

The notion of universality of number ideas, together with the variety of symbols used to represent these ideas, should be reinforced as a result. A deeper appreciation of the advantages of the Hindu-Arabic place-value system should also be an outcome of research and discussion.

SECTION I

Skills and understandings set out in Section H, and in all previous sections, are to be maintained.

Counting activities may be extended to include numbers to bases other than ten. (Refer to the discussion on Place Value in this Guide (see page 14) for a fuller account of these numbers.) These activities could be taken in conjunction with the enrichment activities discussed under Place Value. For example :

Replace the stars with appropriate numerals :

(i) To base 4 : 1, 2, 3, 10, 11, 12, 13, 20, * * *

(ii) To base 8 : 3, 6, 11, 14, 17, 22, 25, * * *

Counting activities with numbers to base 10 may include combinations of operations as in the following examples :

(i) (0, 1) ; (1, 3) ; (2, 5) ; (3, 7) ; (*, *) ; (*, *).

In this example the second of the pair may be found by multiplying the first number by 2 and adding 1.

(ii) (1, 2) ; (3, 10) ; (5, 26) ; (7, *) ; (9, *).

Here the second number is the square of the first, plus 1.

(iii) ($\frac{1}{4}$, $\frac{1}{8}$) ; ($\frac{1}{2}$, $\frac{1}{2}$) ; (1, 2) ; (2, 8) ; (3, *) ; (4, *).

The second number is seen to be twice the square of the first.

As has been suggested in earlier sections, children should be encouraged to create pattern problems for other children to solve.

Prime Numbers and Factors

An examination of numbers in terms of their composition may be undertaken. It will be discovered that numbers, other than 1, are found to contain two or more different factors. This property can be demonstrated diagrammatically by representing numbers by arrays of dots. Some numbers can be represented by many different rectangular arrays, others by only one single row or column. The latter are called prime numbers. For example, the number symbolised by 5 can be shown as • • • • •, whereas 12 can be represented by :

• • • • • • • • • • • • or

• • • • • •
• • • • • • or

• • •
• • •
• • •
• • •

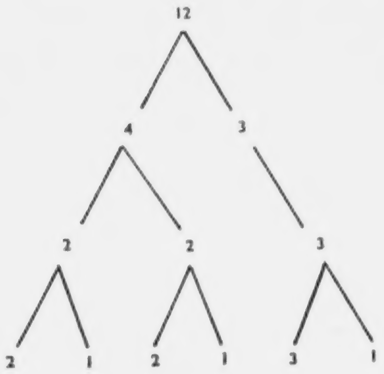
Hence 5 = 5 × 1 (or 1 × 5) and thus has two factors, while

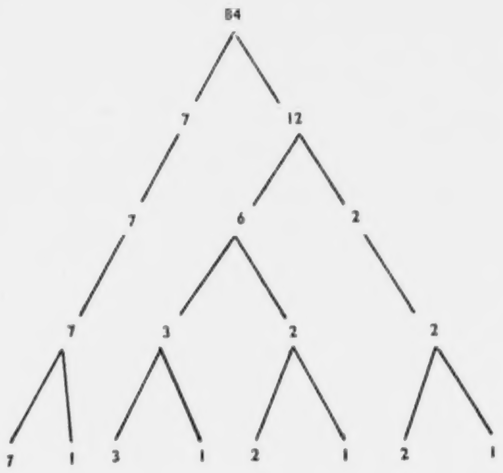
12 = 12 × 1
= 6 × 2
= 4 × 3

and thus is seen to have six factors—12, 1, 6, 2, 4, 3.

A prime number may be defined as a whole number which has two different integral factors, itself and one. Other whole numbers (composite numbers) can be renamed as multiples of prime numbers. For example, 12 = 3 × 2 × 2 × 1. (It should be realised that the number one is unique in that it is neither prime nor composite—it does not have two different factors that are whole numbers.)

Factor trees can be constructed to illustrate the composition of composite numbers. For example :





Figurate Number

A preliminary investigation into figurate number was suggested in the Guide to Section G (see page 8), where square and triangular numbers were considered and a link between odd and square numbers was shown. Further investigation into the properties of figurate number, odd and even numbers, and prime and composite numbers may well prove profitable and interesting to the child.

History of Number

Children who have developed an interest in the history of number can pursue this interest further into a study of Egyptian and Arabian systems. The development of the concept of number occurred as the need for extending the system was felt. This idea can provide a theme about which direction can be given to the study of the history of number.

PLACE VALUE

SECTION H

The pattern of decimal notation should be familiar to children who have completed Section G. Regular exercises should be employed to ensure that this understanding is maintained. In Section H the range of numbers used is extended to the limits ·001 and 10,000,000.

Exercises such as the following will be found useful :

1. Arrange in order from the smallest to the largest :
   - (*a*) 100,000 ; 99,900 ; 99·999 ; 111·1
   - (*b*) ·500 ; 1·01 ; ·050 ; ·49 ; ·51
   - (*c*) $\frac{3}{100}$; 3·0 ; ·30 ; 3·3 ; ·333.
2. In the numeral 138·704 the 8 stands for a number which is (choose the appropriate answer from (*a*), (*b*), and (*c*)) :
   - (*a*) twice the value of the 4
   - (*b*) two hundred times the value of the 4
   - (*c*) two thousand times the value of the 4.
3. Using structured material such as Dienes's Multibase Arithmetic Blocks, base 10, choose material to represent 1·305.

4. How many bundles, each of 1,000 sticks, can be made from 10,000,000 sticks ?

5. How many cupfuls of milk, each ·05 of one gallon, can be obtained from a tank holding 500 gallons ?

In order to appreciate the advantages of the modern Hindu-Arabic notation it may be useful to examine and discuss some systems in which place value had no part—for example, the Egyptian and the Roman systems. Simple computations using these systems should clearly emphasise their limitations. The importance of zero should be well understood. The invention of the symbol and its use had an important and interesting effect on subsequent mathematics development.

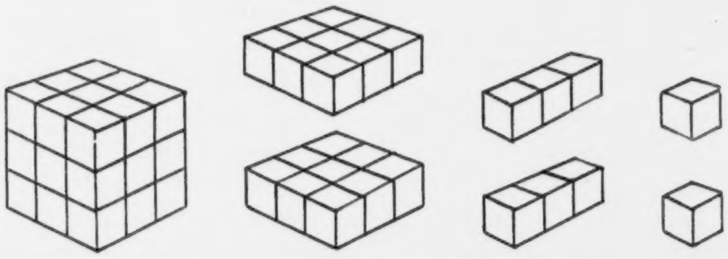
SECTION I

Knowledge and understanding of the decimal system of place value should be maintained and extended.

A deeper understanding may well result from a study of systems to bases other than 10. Structured aids are available or may be constructed to assist in this study.

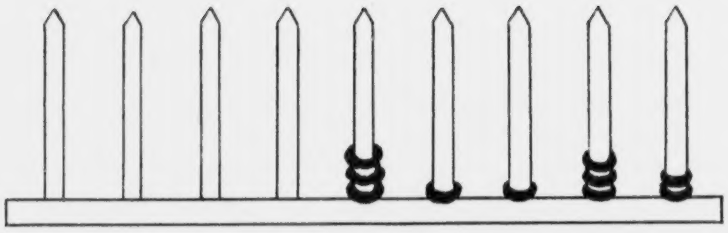
Dienes's Multibase Arithmetic Blocks provide one such aid. This material was referred to in the Guide to Section G, where the use of base 10 blocks was suggested as a means of gaining understanding in formal processes. In Section I the child can investigate numbers written to, say, base 3. He can physically combine elements of the material to illustrate numerals. He can vary the block chosen to represent unity in order to illustrate "tricimals", "bicimals", or other numerals. Just as a numeral such as 5·75 to base 10 is called a decimal, so the numeral 2·12 to base 3 may be called a "tricimal", and the numeral 1·01 to base 2 may be a "bicimal".)

Consider base 3 material. The selection shown below may represent 1222, 1·222, 122·2, ·1222, or 12·22, depending on which piece is chosen to represent 1.



Place value is of no consequence in selecting the material. It is only when the number is recorded that a convention of place value serves such a valuable purpose.

An aid which uses a place value convention is the spike abacus. Such an abacus is described (for base 10) in the Guide to Section G (see Addition, page 32).



In the above diagram the number 31,132 is illustrated. This numeral communicates little unless information is given as to which base is being used. However, a convention applies. If no indication is given as to which base is being used, we may assume that the numeral is to base *ten*. If a base other than *ten* is intended, then an indication such as the following may be used :

384_{nine} $\quad$ $212{\cdot}11_{\text{three}}$ $\quad$ $421{\cdot}3_5$

Although it is desirable for children to recognise both forms of notation, it is preferable, at least in the primary school, for the word form to be used. (It is interesting to note that the numeral for nine to the base *nine* would be written as 10 in figures. Similarly, to bases *three* and *five* respectively, three = 10, five = 10.)

When numerals other than those written to base *ten* are verbalised, care must be taken not to use the words hundred, thousand, and so on. For example, the numeral $21122{\cdot}2_{\text{three}}$ may be read as "two, one, one, two, two, point two", **not** as "twenty-one thousand, one hundred and twenty-two point two."

In base 4 ; 31132 represents $3 \times (4 \times 4 \times 4 \times 4) + 1 \times (4 \times 4 \times 4) + 1 \times (4 \times 4) + 3 \times (4) + 2 \times (1)$.

In base 5 ; 31132 represents $3 \times (5 \times 5 \times 5 \times 5) + 1 \times (5 \times 5 \times 5) + 1 \times (5 \times 5) + 3 \times (5) + 2 \times (1)$.

Examples :

1. Rewrite the following numerals in base *three* :
   1 ; 5 ; 29 ; 109.
2. Rewrite the following numerals in base *ten* :
   3_{five} $\quad$ 124_{five} $\quad$ 3404_{five} $\quad$ 20002_{five}
3. (*a*) $201_{\text{three}} + 222_{\text{three}} + 1002_{\text{three}} = \square_{\text{three}}$
   (*b*) $2112_{\text{three}} - 1010_{\text{three}} = \square_{\text{three}}$
   (*c*) $12_{\text{three}} \times 2_{\text{three}} = \square_{\text{three}}$
   (*d*) $2121_{\text{three}} \div 2_{\text{three}} = \square_{\text{three}}$
4. Counting exercises (see Pattern and Order in the Number System, Section 1).

Indicial notation as an alternative way of recording repeated factors may be introduced.

Hence :

$$5 = 5^1, \quad 5 \times 5 = 5^2, \quad 5 \times 5 \times 5 = 5^3, \text{ and so on.}$$

In base *ten*, successive places may be denoted as follows :

• • •	10^4	10^3	10^2	10	Units	$\frac{1}{10}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	• • •

Similarly for other bases, for example, base *three* :

• • •	3^4	3^3	3^2	3	Units	$\frac{1}{3}$	$\frac{1}{3^2}$	$\frac{1}{3^3}$

In this case (base *three*) the numerals 3 and 4 should be written as 10 and 11. Thus the above example may be more appropriately written as :

• • •	10^{11}	10^{10}	10^2	10	Units	$\frac{1}{10}$	$\frac{1}{10^2}$	$\frac{1}{10^{10}}$

Examples :

(i) $3413 = 3 \times 10^3 + 4 \times 10^2 + 1 \times \square + 3 \times \square$

(ii) $3413_{\square} = 3 \times 5^3 + 4 \times 5^2 + 1 \times 5 + 3 \times 1$

(iii) $3413_{\text{eight}} = 3 \times \square^3 + 4 \times \square^2 + 1 \times \square + 3 \times \square$

(iv) $3413 = \square_{\text{five}}$

The last example may be solved by renaming 3413 in powers of five.

$$\begin{aligned} 3413 &= 3125 + (125 \times 2) + 25 + (5 \times 2) + 3 \\ &= 5^5 \times 1 + 5^3 \times 2 + 5^2 \times 1 + 5 \times 2 + 3 \times 1 \\ &= (1 \times 5^5) + (0 \times 5^4) + (2 \times 5^3) + (1 \times 5^2) + (2 \times 5) \\ &\quad + (3 \times 1) \end{aligned}$$

then $3413_{\text{ten}} = 102123_{\text{five}}$

To rename 3413_{ten} in base *five* the following procedure may be adopted :

Build up a table showing powers of 5 and break up 3413 as $(3125 + 288) = (3125 + 125 \times 2 + 25 + 5 \times 2 + 3)$

5^5 (3125_{ten})s	5^4 (625_{ten})s	5^3 (125_{ten})s	5^2 (25_{ten})s	5 (5_{ten})s	Ones
1	0	2	1	2	3

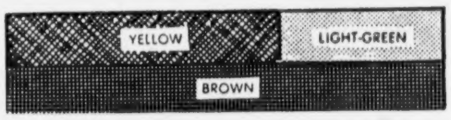
Hence $3413_{\text{ten}} = 102123_{\text{five}}$.

EQUATIONS

SECTION H AND SECTION I

Equations and inequations (statements involving inequalities) occur as the symbolic representations of problems involving number. Children meet these problems in all topic areas of the mathematics course. *Equations should not be treated in isolation from other topics*; rather they should be regarded as useful means of recording given information. Manipulation of the elements of an equation to find the solution to an unknown quantity should be a meaningful rather than a rote procedure. To this end the child should be able to relate equations to concrete situations and to write simple equations to record given information. In Section H there is little justification for setting a child long, complex equations to solve.

In Section C the child could relate concrete situations to written equations and create equations which would adequately explain a situation he observed. For example, given the rod pattern illustrated below and told that the measure of white is 1, he could write $5 + 3 = 8$. In Section H the same pattern may be presented, but now the value of red may be given as $0{\cdot}1$. He may now write the equation $0{\cdot}25 + 0{\cdot}15 = 0{\cdot}4$.



Here, although the same simple form of the equation is used as was used in Section C, the child's development of understanding and skill in value relations is evident.

Opportunities to record situations by equations (and inequations) should be given frequently. For example, in weighing activities (using a pound mass as the unit) a child may record :

$$1 + \tfrac{1}{8} \times 5 + \tfrac{1}{16} \times 6 = 2$$

or $1 + 5 \times \tfrac{1}{8} + 5 \times \tfrac{1}{16} < 2$

or he may be challenged to solve

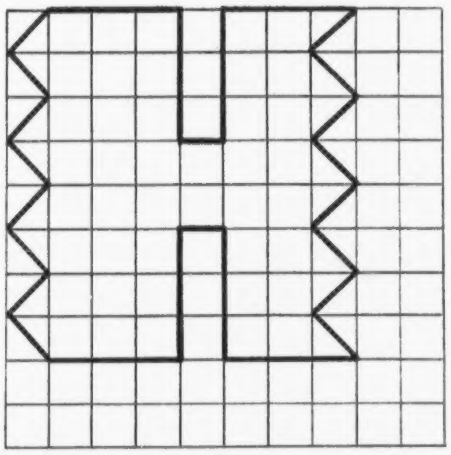
$$1 + \tfrac{1}{8} \times \square + \tfrac{1}{16} \times 6 = 2$$

Similar opportunities arise in many measurement activities. In volume (Section G), block constructions were described in recorded equations. Children should be encouraged to record their observations from various points of view. For example, a region with an area of 50 squares (as illustrated) may be viewed as :

(i) $64 - [8 \times \tfrac{1}{2} + 8 \times \tfrac{1}{2} + 3 \times 2] = n$

(ii) $2 \times (7 + 6 + 6 + 6) = n$
or $2 \times 7 + 3 \times 6 + 3 \times 6 = n$

(iii) $4 + 8 + 8 + 8 + 2 + 8 + 8 + 4 = n$
and so on.



Children should be given frequent opportunities to encode verbal problems ; that is, to express the mathematical content of verbal problems in equation form. They should also be encouraged to decode equations by fitting a suitable problem to a given equation. The child has created simple stories to fit equations since Section C. In the example that follows, simple numbers are used so that the decoding will not be inhibited by computation difficulties. Given the equation $7 \times 13 + 11 = 102$, a child may suggest :

> " In each of seven boxes there are thirteen silkworms. Another box contains eleven. Altogether there are $(7 \times 13 + 11)$ or 102 silkworms."

The child may be given the problem of discovering the number of days in the first ten months of the year 1969. He may encode this by writing :

$$365 - (30 + 31) = \square$$

or $$30 + 30 + 30 + 31 + 31 + 31 + 31 + 31 + 31 + 28 = \square$$

or $$3 \times 30 + 6 \times 31 + 28 = \square$$

Equations will be used frequently in number pattern activities such as Find the Rule. Children are asked to find a rule which relates two numbers to each other. For example : Examine the following pairs of numbers and attempt to discover a rule which will enable the second number to be found from the first of each pair :

$$(2, 7)\ ; \qquad (5, 16)\ ; \qquad (7, 22)$$

$$\underline{2} \times \boxed{3} + 1 = \underline{7} \quad \underline{5} \times 3 + 1 = \underline{16} \quad \underline{7} \times 3 + 1 = \underline{22}$$

Some children may write : $\square \times 3 + 1 = \triangle$

An inequation can be considered as a true statement of inequality. The terms " is greater than " (>) and " is less than " (<) replace the " is equal to " (=) of an equation. The solution of inequations is a valuable activity in both the maintenance and the consolidation of number facts. For example :

(i) Find the largest whole number, in each case, which satisfies the following inequations :

(*a*) $8 \times \square < 60$
(*b*) $17 \times \square < 165$
(*c*) $363 \div \square > 18$
(*d*) $996 \div 45 > \square$
(*e*) $317 + \square < 329$
(*f*) $218 - 209 > \square$
(*g*) $6 \times \square < 6$
(*h*) $718 - \square > 0$

(ii) Write fractions for which the following are true :

(*a*) $3\frac{1}{2} - \square > 3\frac{1}{4}$
(*b*) $\frac{1}{3} + \square < \frac{1}{2}$
(*c*) $0{\cdot}25 > 0{\cdot}05 + \square$

(iii) What is the largest fraction you can think of which satisfies the inequation $9 + \square < 10$? This question should provoke discussion.

FORMAL PROCESSES

SECTION H

In this section the child's understanding of, and skill in using, the operations of addition, subtraction, multiplication, and division should be extended. Larger numbers will, on occasions, be used—

totals to 100,000 ;
minuends to 100,000 ;
multiplicands to 10,000 ;
multipliers to 100 ;
dividends to 10,000 ;
divisors to 100.

Short division is to be introduced. Computational limits are dividends to 1,000 and divisors to 12.

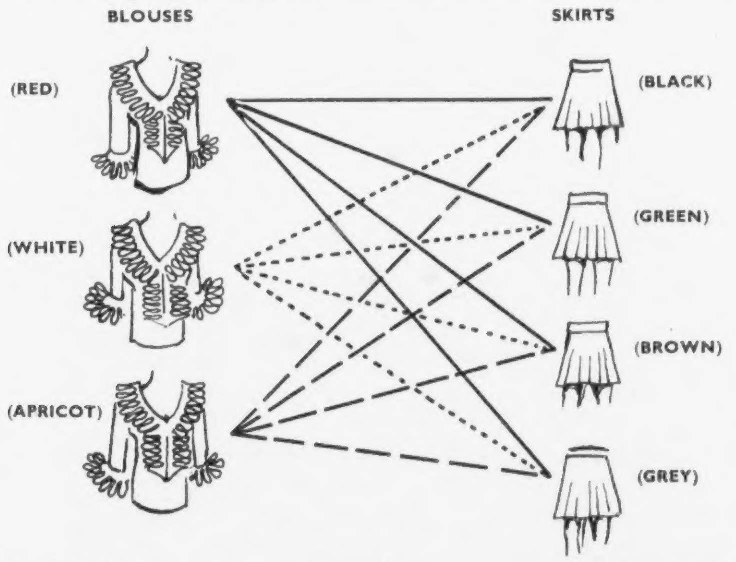
MULTIPLICATION

The child understands multiplication as repeated addition. He has also studied this operation as the inverse of division. He has probably linked multiplication with rectangular arrays.

Another interpretation of the operation may be that of the association of a pair of numbers called factors with a unique number called the product. Hence, for example, the numbers 3, 4 would be associated with the number 12. Given the number triple (3, 4, 12) the child might write :

(*a*) $3 \times 4 = 12$ (*c*) $12 \div 3 = 4$ (*e*) $3 + 3 + 3 + 3 = 12$
(*b*) $4 \times 3 = 12$ (*d*) $12 \div 4 = 3$ (*f*) $4 + 4 + 4 = 12$

Another understanding of 12 as the product of 3 and 4 comes out of a situation such as the following : A girl has three blouses and four skirts. How many combinations of blouse and skirt can she choose ?



red—black
red—green
red—brown
red—grey

white—black
white—green
white—brown
white—grey

apricot—black
apricot—green
apricot—brown
apricot—grey

Thus the girl can choose 3 × 4 (or 12) blouse—skirt combinations.

The child should also see that, by first taking a skirt, the girl can make three choices as to the blouse she will wear. The *total* number of pairings she can make is 4 × 3 (or 12).

Multiplication by 0 and by 1 has meaning in this context. If the child has no (0) skirts she can have no pairings with skirts, no matter how many blouses she might have. Hence, for example, $3 \times 0 = 0$. If the girl has 1 skirt she can make 4 pairings if she has 4 blouses. Hence $1 \times 4 = 4$, also $4 \times 1 = 4$.

LONG DIVISION

Teachers should refer to the Curriculum Guide, Section G, for a detailed examination of this topic. In Section H a slight change in the lay-out of the algorithm can take place, and the size of the numbers used can be increased.

The child should now have considerable skill in estimating quotients, a skill that needs maintenance, and it is only a small step for him to adopt the traditional form of the long division algorithm. For example :

$$\begin{array}{r|r|l} 79 & 1078 & \\ & 790 & 10 \\ \cline{2-2} & 288 & \\ & 237 & 3 \\ \cline{2-3} & 51 & 13 \\ \cline{3-3} \end{array}$$

Answer : 13, rem. 51

$$\begin{array}{r|r|} & 13 \\ & 3 \\ & 10 \\ \hline 79 & 1078 \\ & 790 \\ \cline{2-2} & 288 \\ & 237 \\ \cline{2-2} & 51 \end{array}$$

Answer : 13, rem. 51

$$\begin{array}{r|r|} & 13 \\ \hline 79 & 1078 \\ & 790 \\ \cline{2-2} & 288 \\ & 237 \\ \cline{2-2} & 51 \end{array}$$

Answer : 13, rem. 51

SHORT DIVISION

In the Victorian primary school mathematics course the process of division has been developed through an understanding of division as related to subtraction ; that is, through the idea of successive subtraction. This approach to division leads to the long division method. (See Curriculum Guide, Section G, page 47.)

In Section H, short division is introduced and the child should master the process in this section. Short division utilises the distributive property of division over addition.

To undertake this work successfully, the child should have—

- (*a*) understanding of the operation of division (as developed in earlier sections) ;
- (*b*) understanding of, and skill with, place value and the renaming of numbers ;
- (*c*) an understanding of the distributive property of—
  - (i) multiplication over addition,
  - (ii) division over addition,
  - (iii) division over subtraction ;
- (*d*) ability to name multiples of numbers ;
- (*e*) automatic response in division facts to 144.

Activities such as the following can be undertaken by the child to develop his skill in handling multiples :

(*a*) Rename these numbers using only multiples of 9 :

36	□ + △
63	▽ + ◇
99	○ + ▽

(*b*) Rename these numbers using only multiples of 4 :

48	□ + ▽
124	△ + □
156	◇ + ○

(*c*) Complete the tables below :

(i)

245 ÷ 5	20 ÷ 5 = □ 200 ÷ 5 = □ 45 ÷ 5 = □

(ii)

368 ÷ 4	32 ÷ 4 = □ 320 ÷ 4 = □ 48 ÷ 4 = □

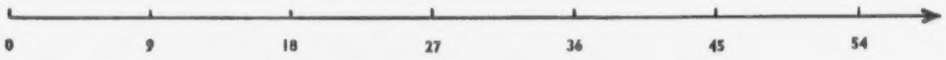
(*d*) Which of the following are the most suitable sets of multiples to solve the given equation ?

	Set A	Set B	Set C
252 ÷ 4 =	240 ÷ 4 = ☐ 48 ÷ 4 = ☐	280 ÷ 4 = ☐ 12 ÷ 4 = ☐	240 ÷ 4 = ☐ 12 ÷ 4 = ☐
414 ÷ 6 =	360 ÷ 6 = ☐ 48 ÷ 6 = ☐ 36 ÷ 6 = ☐	360 ÷ 6 = ☐ 36 ÷ 6 = ☐ 6 ÷ 6 = ☐	360 ÷ 6 = ☐ 48 ÷ 6 = ☐ 6 ÷ 6 = ☐

In developing the topic and in selecting examples of short division, care should be taken so that only one difficulty is introduced at a time.

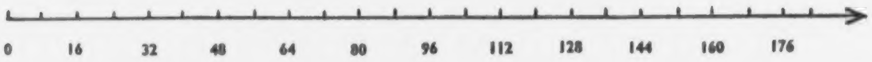
Short division should be understood ultimately through the distribution of division over addition. Such understandings will best come from a variety of suitable introductory experiences of the kind set out below.

(*a*)



(i) How many nines in 18 ?
(ii) How many nines in 36 ?
(iii) How many nines in (18 + 36) ?
(iv) How many nines in 9 ?
(v) How many nines in 45 ?
(vi) How many nines in (45 + 9) ?
(vii) In 54 there are ...... nines.

(*b*)



(i) How many eights in 160 ?
(ii) In 16, how many eights are there ?
(iii) How many eights in (160 + 16) ?
(iv) In 176 there are ...... eights.
(v) In 96, how many eights are there ?

(vi) How many eights in 80 ?

(vii) How many eights in $(96 + 80)$?

(viii) $(96 + 80)$ is another name for ....

(ix) How many eights in 176 ?

(c) Solve each of the following examples a number of ways, each time renaming the dividend by using different multiples of the divisor (as illustrated below) :

(i) $36 \div 3 = \square$

$$\begin{aligned} &(12 + 12 + 12) \div 3 \\ &= (12 \div 3) + (12 \div 3) + (12 \div 3) \\ &= 4 + 4 + 4 = 12 \end{aligned}$$

$$\begin{aligned} (30 + 6) \div 3 &= (30 \div 3) + (6 \div 3) \\ &= 10 + 2 = 12 \end{aligned}$$

$$\begin{aligned} &(30 + 3 + 3) \div 3 \\ &= (30 \div 3) + (3 \div 3) + (3 \div 3) \\ &= 10 + 1 + 1 = 12 \end{aligned}$$

(ii) $984 \div 8 = \square$

$$\begin{aligned} &(800 + 160 + 24) \div 8 \\ &= (800 \div 8) + (160 \div 8) + (24 \div 8) \\ &= 100 + 20 + 3 \\ &= 123 \end{aligned}$$

$$\begin{aligned} &(800 + 80 + 80 + 24) \div 8 \\ &= 100 + 10 + 10 + 3 \\ &= 123 \end{aligned}$$

The child should be encouraged to find other ways of gaining solutions. These various ways should be discussed and compared by the children to find the easiest method in any given situation.

The child might suggest reasons as to why short division is normally restricted to the use of divisors from 1 to 12.

Dividends can be renamed using subtraction :

$$\begin{aligned} 686 \div 7 &= (700 - 14) \div 7 \\ &= (700 \div 7) - (14 \div 7) \\ &= 100 - 2 \\ &= 98 \end{aligned}$$

In earlier examples the child may utilise extended notation but, as his skill develops, he will move to the traditional algorithm for short division.

Example (i) : 482 ÷ 2 = ☐

$$\begin{array}{rl} 2\;\underline{)400 + 80 + 2} & \qquad 2\;\underline{)482} \\ 200 + 40 + 1 & \qquad\quad 241 \\ 241 & \end{array}$$

$$482 \div 2 = 241$$

Example (ii) : 652 ÷ 4 = ☐

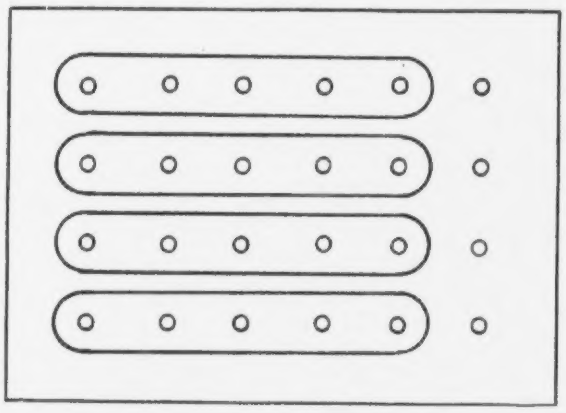
$$\begin{array}{rl} 4\;\underline{)400 + 240 + 12} & \qquad 4\;\underline{)6^{2}5^{1}2} \\ 100 + \;\;60 + \;\;3 & \qquad\quad 1\;6\;3 \end{array}$$

$$652 \div 4 = 163$$

In earlier stages examples will not involve remainders. Later, remainders should occur. They might be introduced through the use of the number line. Discrete objects might also be used.

Example (i) : 24 ÷ 5 = ☐

How many collections of five can be made from 24 objects ?

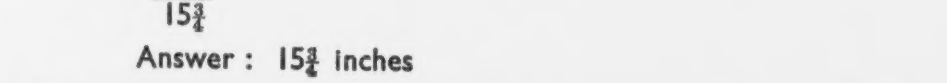


Are all the objects accounted for ?

How many are left over ?

24 = (4 × 5) + 4.

Example (ii) : 214 ÷ 12 = ☐



$$\begin{array}{l} 12\;\underline{)214} \\ \qquad 17,\ \text{remainder } 10 \end{array}$$

Some problems will require total division of the dividend and may consequently involve a fraction in the quotient.

Example : A wire 63 inches long is to be divided into four equal parts. How long is each part ?

$$\begin{array}{l} 4\underline{)63} \\ \;\;15\tfrac{3}{4} \end{array}$$

Answer : $15\frac{3}{4}$ inches

Each teacher will use the sequence he finds most useful. A suitable sequence of difficulties is as follows :

3) 9
3) 39
3)693
} no regrouping

3)45
3)342
3)522
} regrouping

3) 0
3) 30
3)309
3)390
} zeros

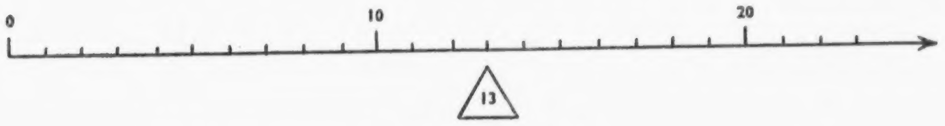
3) 10 Answer : 3, remainder 1 ; or $3\frac{1}{3}$
3) 40 (*Note* : Statement of answer will depend on problem from which the division exercise arose.
3)695 In practice examples, the form of answer required should be made clear.)
3) 47
3)343
3)524
} Remainders

When the child has mastered the complexities involved in regrouping, the use of zeros, and remainders, larger numbers can be progressively introduced.

ROUNDING OFF

Examples :

(*a*) Use the number line shown to help answer the following questions :



(i) Is 13 closer to 10 or to 20 ?

(ii) Which would be the better approximation to 13 ?

(iii) Which number, 20 or 30, would be the better approximation for 28 ? Why ?

(*b*) Round each of the following to the nearest 100 :
324 ; 210 ; 451.

(Note—Children should discuss to which of two given numbers a number should be rounded when it is half-way between the two given numbers. For example : Should 85 be rounded to 80 or 90 ? Should 350 be rounded to 300 or 400 ?

Generally, in such circumstances the number is rounded up, so 85 would be rounded to 90, and 350 would be rounded to 400.)

(*c*) Use rounded numbers to check answers :

The example	247 + 169 = □	306 − 141 = □
Numbers rounded to nearest hundred	200 + 200 = □	300 − 100 = □
Approximate answer	▭	▭
Actual answer	▭	▭

(Examples of children's computation work could be discussed.
77 + 31 = □. Anne's answer was 108.

She checked it by rounding off the numbers to the nearest ten.
80 + 30 = 110.

Was Anne's answer reasonably close to the estimated answer ?

484 ÷ 8 = □. Mark's answer was 58.

He rounded off and obtained 500 ÷ 10 = 50.

Was his answer reasonably close to the estimated answer (50) ?

Was Mark's first answer a plausible answer ?)

(*d*) Examine the size of the differences between actual and estimated answers (see table below).

Statement	Rounded statement	Approximate answer	Exact answer	Difference between approximate answer and exact answer
117 × 8 = □	100 × 10 = □	1000	936	64
481 × 9 = □	500 × 10 = □	5000	4329	671
341 + 214 = □	300 + 200 = □	500	555	55
506 − 191 = □	500 − 200 = □	300	315	15
352 ÷ 8 = □	400 ÷ 10 = □	40	44	4

Suggest reasons to explain why some of the differences are large and others are small.

ERROR DETECTION

The child has gained reasonable mastery in solving computational exercises involving whole numbers. It is appropriate in this section for him to apply error detection methods to his answers. For each method the child should observe the pattern of its working and, perhaps, be interested to apply it.

The child is familiar with properties of both odd and even numbers and with the use of inverse operations in providing checks for computation. His attention should be drawn again to these checking devices.

PROBLEMS (Section H and Section I)

Children should be given opportunities to use mathematical understandings and skills in solving problems (verbal and practical). Reading difficulties should be kept to a minimum and computational difficulties restricted to realistic situations that are meaningful and interesting to the child.

The following variations might be used :

(*a*) Matching a problem to a mathematical statement :

Eighteen pieces of leather are sewn together to make a basket-ball. How many pieces would have to be cut to make 98 such basket-balls ?

$$\begin{array}{c}\text{Choose the statement which}\\ \text{matches the problem}\end{array}\left\{\begin{array}{l} 98 \times 18 = 1764 \\ 98 + 18 = \ \ 116 \\ 98 \div 18 = 5\tfrac{8}{18} \\ 98 - 18 = 80 \end{array}\right.$$

(*b*) Writing a mathematical statement to fit a situation :

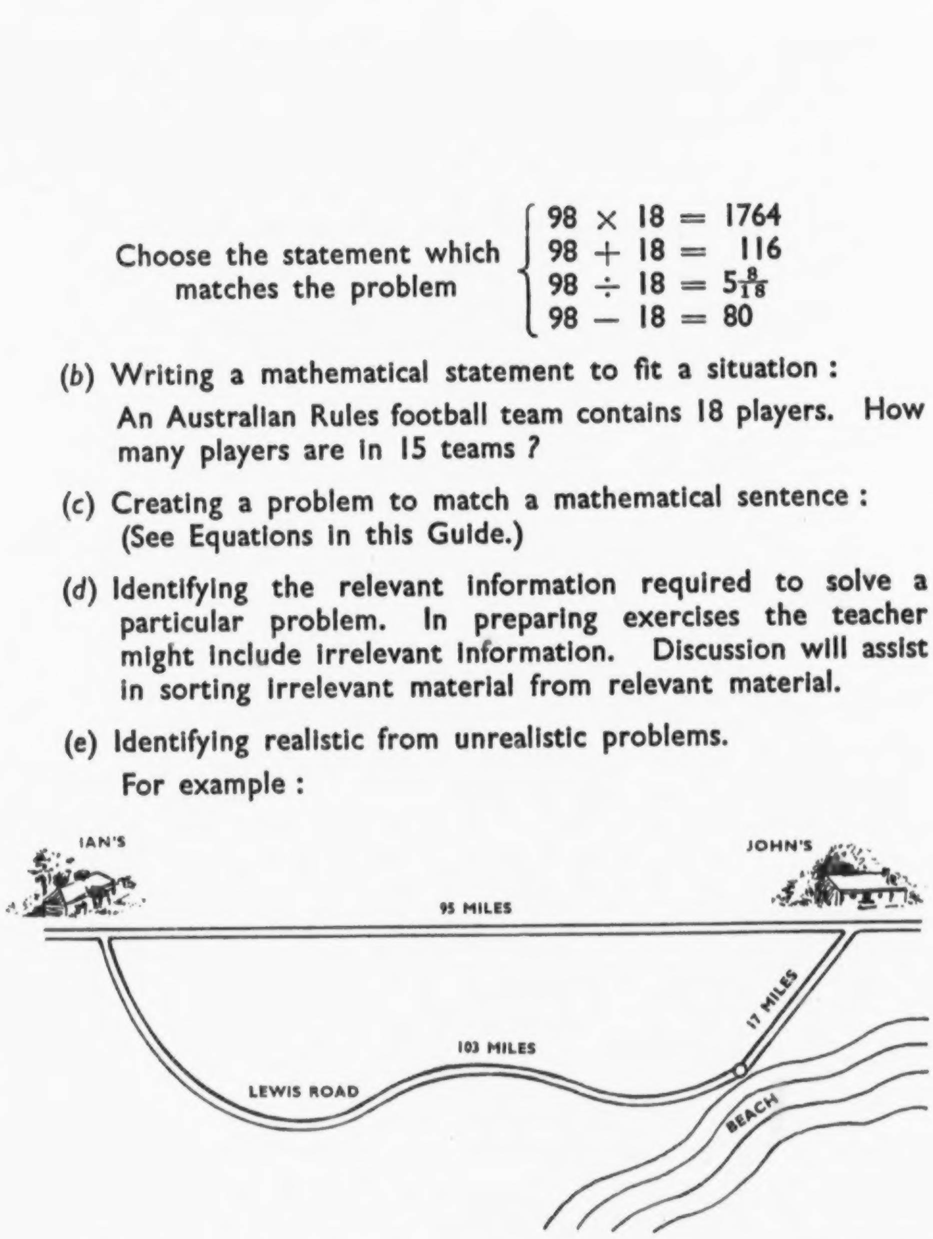
An Australian Rules football team contains 18 players. How many players are in 15 teams ?

(*c*) Creating a problem to match a mathematical sentence : (See Equations in this Guide.)

(*d*) Identifying the relevant information required to solve a particular problem. In preparing exercises the teacher might include irrelevant information. Discussion will assist in sorting irrelevant material from relevant material.

(e) Identifying realistic from unrealistic problems.

For example :



(i) Last Saturday Stuart walked from Ian's house to John's house and then to the beach. He walked home along Lewis Road. How far did he walk ?

(ii) John lives 17 miles from the beach. Ian lives 103 miles from the beach (as shown in the diagram). How far is John's house from Ian's.

In the first example the problem is unrealistic in terms of the likelihood of the situation. In the second problem insufficient data are given for a unique answer to be obtained. The child should be given opportunity to discuss such problems.

SECTION I

Understandings and skills the child has developed in previous sections should be maintained. Tables and number facts can be involved in regular pattern exercises. Processes might be made more complex, and variety might be introduced by involving interrelationship of operations, as in the following examples :

Find numbers which complete the exercises :

(i)
$$\begin{array}{r} **2 \\ 7*\,\overline{)7956} \\ **** \\ \hline *** \\ *** \\ \hline 0 \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} 79\cdot5 \\ \times\quad * \; * \\ \hline ** \; * \\ ** \; * \\ \hline 87\cdot45 \\ \hline \end{array}$$

(iii)
$$\begin{array}{r} 72\cdot85 \\ \cdot*0 \\ \nabla3\cdot1* \\ 4\nabla*\cdot91 \\ \hline 519\cdot02 \\ \hline \end{array}$$

(iv)
$$\begin{array}{r} 6008 \\ **1* \\ \hline 22*3 \\ \hline \end{array}$$

(v) $43\cdot9 - 3 \times * = 1$

(N.B. * used in these examples denotes a missing digit. It may stand for the same or different digits. Where two identical digits are indicated the frame ▽ is used.)

The child has had experience in rounding off in the estimation of answers and in error detection. He should continue to employ this technique for these purposes. Another technique that might interest him in Section I is that involving digital roots.

A digital root is obtained by adding the digits of a numeral until only one digit results. For example :

$$\begin{array}{l} 97 \\ \swarrow \;\; \searrow \\ 9 + 7 = 16 \\ \qquad\quad \swarrow \;\; \searrow \\ \qquad\quad 1 + 6 = \underline{7} \end{array} \qquad\qquad \begin{array}{r} 347 \\ 3 + 4 + 7 = 14 \\ 1 + 4 = \underline{5} \end{array}$$

Example : Complete the following table :

Numeral	72	264	192	503	698
Digital Root	9	3	□	△	▽

Consider the following examples :

(*a*)
$$\begin{array}{rl} 237 & \longrightarrow 3 \\ 459 & \longrightarrow 9 \\ 625 & \longrightarrow 4 \\ \hline 1321 & \longrightarrow \boxed{7} \\ \hline \end{array}$$

$3 + 9 + 4 = 16 \longrightarrow \boxed{7}$

(*b*) $234 \times 22 = \square$

$$\begin{array}{rl} 234 & \longrightarrow 9 \\ \times \quad 22 & \longrightarrow 4 \\ \hline 468 & \\ 4680 & \\ \hline 5148 & \longrightarrow 18 \longrightarrow \boxed{9} \end{array} \left.\begin{array}{l} \\ \\ \end{array}\right\} \begin{array}{l} 9 \times 4 = 36 \\ 3 + 6 = \boxed{9} \end{array}$$

If the answer to a computation is correct, the digital roots (in squares as above) will be equal.

FRACTIONS

SECTION H

Aim

To extend the child's understanding of fractions and of operations involving fractions.

A. ADDITION AND SUBTRACTION OF VULGAR FRACTIONS

1. Teachers should refer to Curriculum Guide, Section G, for a suggested sequential development of this aspect of the topic. Section H is concerned primarily with the maintenance of understandings and skills involving addition and subtraction of fractions developed in Section G.

Examples should now include three addends and denominators up to thirty. Children should still be encouraged to find many ways of correctly solving given examples. No one particular method should be developed to the exclusion of others. For example :

First Method	*Alternative Method*
$2\frac{1}{2} + 1\frac{1}{3} = 2\frac{3}{6} + 1\frac{2}{6}$	$2\frac{1}{2} + 1\frac{1}{3} = 2\frac{3}{6} + 1\frac{2}{6}$
$= (2 + \frac{3}{6}) + (1 + \frac{2}{6})$	$= (\frac{12}{6} + \frac{3}{6}) + (\frac{6}{6} + \frac{2}{6})$
$= 2 + (\frac{3}{6} + 1) + \frac{2}{6}$	$= \frac{15}{6} + \frac{8}{6}$
$= 2 + 1 + \frac{3}{6} + \frac{2}{6}$	$= \frac{23}{6}$
$= 3\frac{5}{6}$	$= 3\frac{5}{6}$

2. When grading examples, teachers should consider the following variations :

(i) $\frac{1}{3} + \frac{1}{6} = \square$

(ii) $3 + \frac{1}{6} = \square$

(iii) $3\frac{3}{4} + \frac{1}{8} = \square$

(iv) $2\frac{2}{3} + 1\frac{1}{3} = \square$

(v) $2\frac{5}{8} + 2\frac{1}{2} = \square$

(vi) $3\frac{5}{6} + 2\frac{1}{4} = \square$

(vii) $3\frac{7}{8} + 3\frac{1}{3} + \frac{1}{4} = \square$

(viii) $\frac{7}{8} - \frac{3}{8} = \square$

(ix) $3\frac{5}{6} - \frac{1}{6} = \square$

(x) $2\frac{1}{2} - \frac{1}{3} = \square$

(xi) $2\frac{1}{3} - \frac{1}{2} = \square$

(xii) $3\frac{3}{4} - 1\frac{7}{8} = \square$

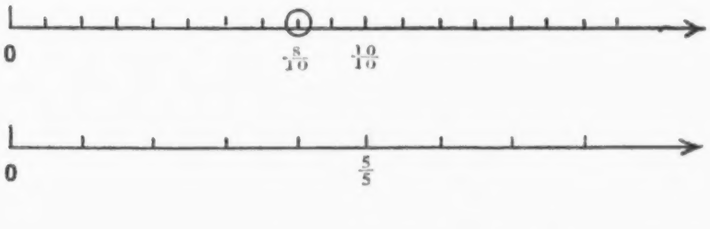
3. No attempt is made to utilise a formal rule for renaming fractions so that they have a common denominator. Number line activities will assist children in this task. In the preparation of exercises which will involve comparison of fractions, or the carrying out of operations between them, denominators should be carefully chosen so that a common denominator can be found readily, either by inspection or by appeal to a number line or to Cuisenaire or other suitable material. For example, $\frac{2}{3}$ and $\frac{7}{9}$ might be compared for size, but to compare $\frac{2}{3}$ with $\frac{7}{11}$ would probably not be appropriate.

$\frac{1}{2}+2\frac{1}{3}=?$

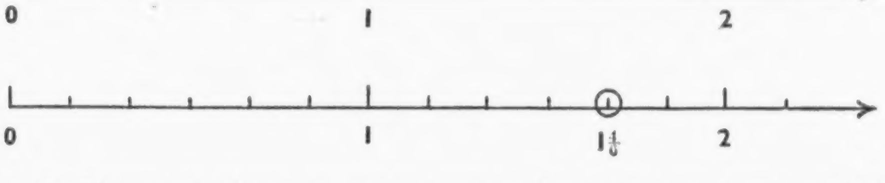
4. In this section children are encouraged to express their answers in the simplest terms. Reducing a fraction to its lowest terms should be treated as an informal activity in renaming the fraction in such a way that the denominator is the smallest whole number possible. Plenty of practice using a set of number lines is invaluable for this purpose. For example :

(*a*) Give a simpler name in each case for the circled point on the number line.

(i)



(ii)



(*b*) Three of these fractions name the same number ; which are they ?

$\frac{1}{4}$, $\frac{3}{12}$, $\frac{1}{3}$, $\frac{5}{20}$.

In order to understand when a fraction is expressed in its lowest terms a child will need to know what is a common divisor. Once he has this knowledge he can be led to see that a fraction is expressed in its lowest terms when there is no divisor (other than 1) common to both numerator and denominator. Thus $\frac{5}{10}$ is not in its lowest terms because 5 is a common divisor for both the numerator and the denominator. Similarly $\frac{9}{6}$ is not in its lowest terms because 3 is a divisor common to both 9 and 6.

The child might be given activities such as the one below to reinforce the idea of common divisors. He should also be encouraged to seek the largest common divisor, since this speeds simplification.

Example : Select the greatest divisor common to 36 and 16 from the following list :

Number	Divisors
36	1, 2, 3, 4, 6, 9, 12, 18, 36
16	1, 2, 4, 8, 16

Once the child has reached the stage where he can rewrite a numerator and a denominator by using factors, the step from, say, $\frac{5}{10}$ to $\frac{1}{2}$, is small. Here he uses his knowledge of the identity property of multiplication (see Curriculum Guide, Section F, page 54) which tells him that the product of 1 and any number leaves the number unchanged. The child knows $\frac{5}{5}$ is another name for 1. Hence

$$\frac{5}{10} = \frac{5 \times 1}{5 \times 2} = \frac{5}{5} \times \frac{1}{2} = 1 \times \frac{1}{2} = \frac{1}{2}$$

Can the fraction, $\frac{2}{8}$, be written in lower terms ?

$$\frac{2}{8} = \frac{2 \times 1}{2 \times 4} = \frac{2}{2} \times \frac{1}{4} = 1 \times \frac{1}{4} = \frac{1}{4}$$

Hence $\frac{2}{8}$ is renamed as $\frac{1}{4}$, with the fraction now in its lowest terms.

Is $\frac{2}{2}$ expressed in lowest terms ?

$$\frac{2}{2} = \frac{2 \times 1}{2 \times 1} = \frac{2}{2} \times 1 = \frac{1}{1} \times 1 = 1$$

Likewise $\frac{7}{7} = 1$

In general $\frac{\square}{\square} = \frac{\square \times 1}{\square \times 1} = \frac{1}{1} = 1$ (for $\square \neq 0$)

We have now reached the traditional point where, say, $\frac{4}{16}$ is seen as :

$$\frac{1\times 4}{4\times 4} = \frac{1}{4}\times\frac{\cancel{4}^{1}}{\cancel{4}_{1}} = \frac{1}{4}$$

5. In earlier sections the child has gained an understanding of the mathematical properties using whole numbers. Section H is the stage where the child relates these properties to fractions. Up until now the child has been using these properties in an intuitive fashion. While rote answers should be avoided, a child should be encouraged to justify in his own words a step he has taken in solving a mathematical problem. Most of the work might best be treated orally.

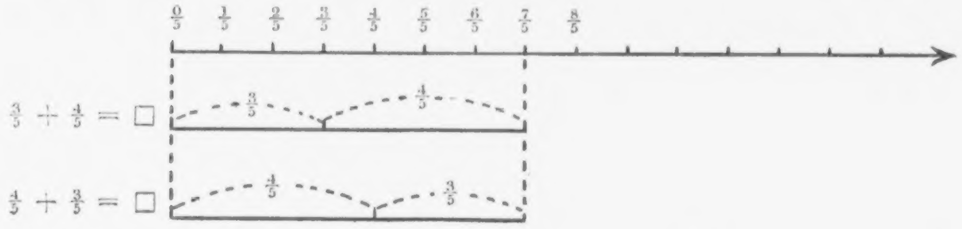
Written examples might take the following forms :

(a) Commutative Property of Addition

(i) Look at the example shown on the number line.

First find the solution for each example.

Does order of addition change the solution ?



$\frac{0}{5}$
$\frac{1}{5}$
$\frac{2}{5}$
$\frac{3}{5}$
$\frac{4}{5}$
$\frac{5}{5}$
$\frac{6}{5}$
$\frac{7}{5}$
$\frac{8}{5}$
$\frac{3}{5} + \frac{4}{5} = \square$
$\frac{3}{5}$
$\frac{4}{5}$
$\frac{4}{5} + \frac{3}{5} = \square$
$\frac{4}{5}$
$\frac{3}{5}$

3 fifths + 4 fifths = 4 fifths + 3 fifths.

(ii) Solve the four equations :

A	B
$\frac{3}{5} + \frac{9}{10} = \square$	$\frac{9}{10} + \frac{3}{5} = \square$
$1\frac{1}{2} + 3\frac{2}{5} = \triangle$	$3\frac{2}{5} + 1\frac{1}{2} = \triangle$

What could you tell about Answer B after having discovered Answer A ?

(iii) Given $\frac{7}{11} + 1\frac{5}{7} = 2\frac{27}{77}$, write and complete

$1\frac{5}{7} + \frac{7}{11} = \square$.

(*b*) *Associative Property of Addition*

The child might be required to complete the following examples :

(i) 5 eighths + (3 eighths + 7 eighths)
= 5 eighths + ☐ eighths
= △ eighths

(ii) (5 eighths + 3 eighths) + 7 eighths
=☐ eighths + 7 eighths
= △ eighths

(iii) $\frac{5}{8} + (\frac{3}{8} + \frac{7}{8})$
$= \frac{5}{8} + \frac{\square}{8}$
$= \frac{\triangle}{8}$

(iv) $(\frac{5}{8} + \frac{3}{8}) + \frac{7}{8}$
$= \frac{\square}{8} + \frac{7}{8}$
$= \frac{\triangle}{8}$

(v) Without making any calculations, match the equation in Column I with the one in Column 2 which has an identical answer.

Column I	Column 2
A. $\frac{5}{8} + (\frac{3}{8} + \frac{7}{8}) = \square$	1. $(\frac{1}{4} + \frac{3}{4}) + \frac{1}{8} = \square$
B. $\frac{1}{4} + (\frac{3}{4} + \frac{1}{8}) = \square$	2. $(\frac{5}{8} + \frac{3}{8}) + \frac{7}{8} = \square$
C. $(\frac{3}{4} + \frac{5}{6}) + \frac{1}{6} = \square$	3. $\frac{3}{4} + (\frac{5}{6} + \frac{1}{6}) = \square$

(vi) Which of the following equations illustrate the associative property of addition with fractions ?
$(\frac{3}{5} + \frac{1}{5}) + \frac{2}{3} = \frac{3}{5} + (\frac{1}{5} + \frac{2}{3})$
$\frac{4}{7} + (\frac{1}{2} + \frac{3}{7}) = (\frac{6}{7} + \frac{1}{2}) + \frac{1}{7}$
$1\frac{1}{2} + (4\frac{1}{4} + 1\frac{1}{3}) = (1\frac{1}{2} + 4\frac{1}{4}) + 1\frac{1}{3}$

(*c*) *Commutative and Associative Properties of Addition*

Examine each of the steps shown in the worked example below and indicate the property used :

$2\frac{1}{5} + 3\frac{1}{10} = 2\frac{2}{10} + 3\frac{1}{10}$
$= (2 + \frac{2}{10}) + (3 + \frac{1}{10})$ (Renaming)
$= 2 + (\frac{2}{10} + 3) + \frac{1}{10}$ (. . . property)
$= 2 + (3 + \frac{2}{10}) + \frac{1}{10}$ (. . . property)
$= (2 + 3) + (\frac{2}{10} + \frac{1}{10})$ (. . . property)
$= 5 + \frac{3}{10}$
$= 5\frac{3}{10}$

(d) *Identity Property of Addition*

(i) Solve the following equations :

$\frac{5}{6} + 0 = \square \qquad \square + 0 = \frac{5}{6}$

$0 + \frac{5}{6} = \square \qquad \square + \frac{5}{6} = \frac{5}{6}$

$3\frac{4}{7} + 0 = \square \qquad 3\frac{4}{7} + \square = 3\frac{4}{7}$

$0 + 3\frac{4}{7} = \square \qquad 0 + \square = 3\frac{4}{7}$

(ii) How many zeros must I add to $1\frac{3}{4}$ for the total to equal 2 ?

B. MULTIPLICATION OF VULGAR FRACTIONS

I. Work in Section H

In Section G the child has handled multiplication of fractions by whole numbers and multiplication of whole numbers by fractions. The work of Section H is a natural extension of this work and includes the multiplication of a fraction by a fraction.

2. Sequence of Development

The early work in this section will mainly take an oral form. Use will be made of rods, number lines, and area diagrams to solve equations involving the multiplication of fractions by fractions.

The second step is done by recording the observations made from the rods etc. These records can then be examined to establish the relevant patterns which lead to a generalisation.

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$
$$\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

Once the correct generalisation has been discovered, various numerical activities which extend to mixed numbers can be presented.

This step requires the child to examine the basic properties of number as they apply to fractions.

For example : Commutative property of multiplication ;
Identity property . . .

Through the use of some of the basic properties of number the child is led to an understanding of computational simplification.

3. Development of an Appropriate Generalisation

The generalisation that we wish the child to derive from his work is that when multiplying two fractions the two denominators are multiplied to obtain a new denominator, and the two numerators are multiplied to obtain the new numerator.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

This is a generalisation from specific examples. It cannot be said to be proved. It is, however, a defining property for the operation of multiplication as applied to fractions. Nevertheless it is important that the child, by means of carefully structured situations, be led to discover these relations as being plausible and therefore forming a reasonable way of combining fractions.

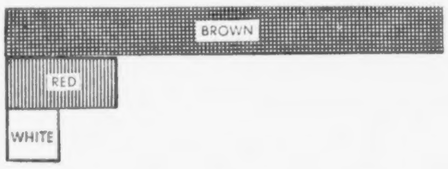
To aid the establishment of this generalisation the child should be presented with numerous examples which make use of number lines, rods, dot arrays, and area diagrams. From this work the child is encouraged to discover the pattern but is *not* required to state it in the formal way as presented above.

The incomplete equation $\frac{1}{2} \times \frac{1}{3} = \square$ can be read as $\frac{1}{2}$ of $\frac{1}{3} = \square$.

Bearing this in mind, the following activities can be investigated:

(*a*) *Rods*: $\frac{1}{2} \times \frac{1}{4} = \square$

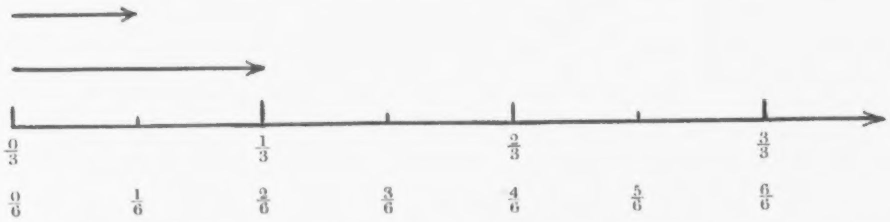
Given that brown is the unit rod, the problem $\frac{1}{2}$ of $\frac{1}{4}$ can be readily solved.



"What is one-quarter of brown?"
"Red is one-quarter of brown."
"What rod is half of this quarter?"
"White is half of this quarter."
"What is the value of white when the value of brown is 1?"
"The white rod is one-eighth."
"Therefore $\frac{1}{2}$ of $\frac{1}{4}$ is equal to $\frac{1}{8}$."

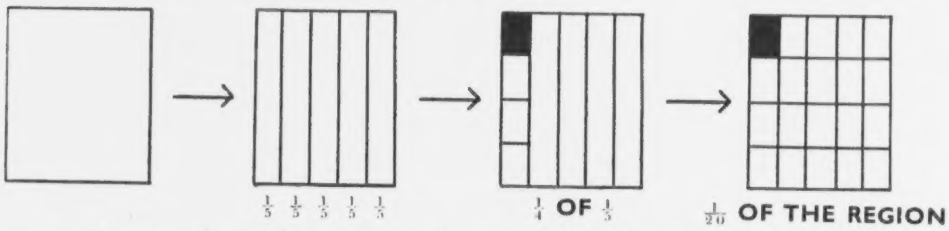
When presenting these examples, teachers will need to select suitable unit rods. In the above example the use of the pink rod as the unit would be unsuitable, since one-quarter of pink is white and there is no rod to show one-half of white.

(*b*) *Number Lines*: $\frac{1}{2} \times \frac{1}{3} = \square$



On the relevant number line presented to the child, one-third can be identified and then a half of this can be found.

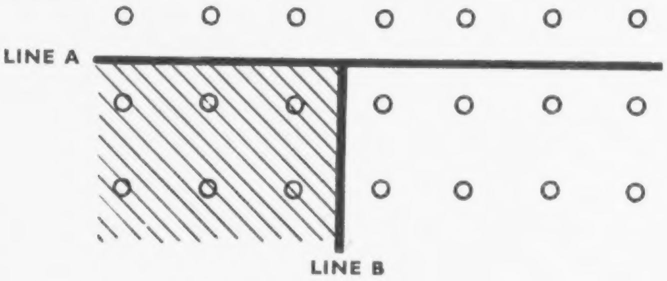
(c) *Area Diagrams* : $\frac{1}{4} \times \frac{1}{5} = \square$


$\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$ $\frac{1}{5}$
$\frac{1}{4}$ OF $\frac{1}{5}$
$\frac{1}{20}$ OF THE REGION

A unit shape has been taken and divided into fifths. One of these fifths has then been divided into quarters, one-quarter of one-fifth being shaded. This shaded section is then related to the shape as a whole.

(d) *Arrays* (*Pegboard*) : $\frac{3}{7} \times \frac{2}{3} = \square$

The unit of 21 pegs or dots is set and the child is asked to use it to solve the given problem. Two-thirds of the array is established. A pencil could be used to indicate this (Line A). The child's attention is then directed to the two-thirds and he is asked to find three-sevenths of this section of the array. This can also be indicated with a pencil (Line B). The isolated section of the array is then related to the whole : $\frac{6}{21}$.


LINE A
LINE B

(e) *Other Activities*

Practical illustrations can be made with fruit if the numerical examples are simple, for example, half of half an orange.

Applied number situations can be used, for example, half of a quarter of one foot ; two-thirds of one-half of a pound of beads.

For some children, the following development may aid in establishing the desired generalisation. It is important to realise that this sequence does not constitute a proof ; it only shows a method that works when the resulting numerals used represent whole numbers. Working from numerous examples in which whole numbers are renamed as fractions, the child will discover the relevant pattern. One example is presented below.

$3 \times 2 = \square$	
$3 \times 2 = 6$	(The child accepts this step from his previous work.)
$\downarrow \quad \downarrow$	
$\frac{9}{3} \times \frac{4}{2} = \frac{36}{6}$	(The factors have been renamed in fractional form.)
$\frac{36}{6} = 6$	(The child realises this from his study of equivalences.)

By multiplying 9 and 4 to give a numerator for the fraction numeral, and by multiplying 3 and 2 to give a denominator for the fraction numeral, the fraction numeral we obtain represents the product of $\frac{9}{3}$ and $\frac{4}{2}$.

The child is now led to the discovery that $\frac{9}{3} \times \frac{4}{2}$ can be rewritten as $\frac{9 \times 4}{3 \times 2}$.

$\frac{9}{3} \times \frac{4}{2} = \frac{36}{6}$ (as above)

Renaming $\frac{36}{6}$ as $\frac{9 \times 4}{3 \times 2}$, we can say that

$$\frac{9}{3} \times \frac{4}{2} = \frac{36}{6} = \frac{9 \times 4}{3 \times 2}.$$

Again, $7 \times 5 = 35$

Renaming 7, 5, and 35 we have : $\frac{21}{3} \times \frac{10}{2} = \frac{210}{6} = \frac{21 \times 10}{3 \times 2}$

Hence $\frac{21}{3} \times \frac{10}{2} = \frac{21 \times 10}{3 \times 2}$.

An understanding of the multiplication of a fraction can develop from the study of a pattern that begins with the familiar multiplication of a whole number by a fraction. For example :

(*a*)

$$\begin{aligned}
12 \times \tfrac{1}{4} &= 3 \ (= \tfrac{3}{1}) \\
6 \times \tfrac{1}{4} &= \tfrac{6}{4} \ (= \tfrac{3}{2}) \\
\tfrac{3}{1} \times \tfrac{1}{4} &= \tfrac{3}{4} = \frac{3 \times 1}{1 \times 4} \\
\tfrac{3}{2} \times \tfrac{1}{4} &= \tfrac{3}{8} = \frac{3 \times 1}{2 \times 4} \\
\tfrac{3}{4} \times \tfrac{1}{4} &= \tfrac{3}{16} = \frac{3 \times 1}{4 \times 4}
\end{aligned}$$

(*b*)

$$\begin{aligned}
24 \times \tfrac{3}{4} &= 18 \\
12 \times \tfrac{3}{4} &= 9 \\
6 \times \tfrac{3}{4} &= \tfrac{9}{2} \\
3 \times \tfrac{3}{4} &= \tfrac{9}{4} = \frac{3 \times 3}{1 \times 4} \\
\tfrac{3}{2} \times \tfrac{3}{4} &= \tfrac{9}{8} = \frac{3 \times 3}{2 \times 4} \\
\tfrac{3}{4} \times \tfrac{3}{4} &= \tfrac{9}{16} = \frac{3 \times 3}{4 \times 4}
\end{aligned}$$

From numerous experiences involving patterns seen in exercises such as the above, the child will freely accept the generalisation expressed by :

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

4. Examples

The following examples are illustrative of the types and the sequences with which these are likely to be experienced :

(*a*) Multiplying a fraction by a fraction, e.g.
$\tfrac{1}{5} \times \tfrac{1}{4} = \square$ to $\tfrac{3}{5} \times \tfrac{3}{4} = \square$.

(*b*) Multiplying a fraction by a mixed number, e.g.
$\tfrac{1}{2} \times 1\tfrac{1}{4} = \square$.

(*c*) Multiplying an improper fraction by an improper fraction, e.g.
$\tfrac{7}{4} \times \tfrac{5}{2} = \square$.

(*d*) Multiplying a mixed number by a fraction, e.g.
$2\tfrac{4}{7} \times \tfrac{3}{5} = \square$.

(e) Multiplying a mixed number by a mixed number, e.g.
$1\tfrac{1}{8} \times 1\tfrac{1}{5} = \square$.

5. Numerical Activities

Teachers should consult the Curriculum Guide, Section G, for examples of the types of activities that, with adaptation, can be used in this section. Others such as the following could be used :

(i) Complete the following table :

Equation	First Factor	Second Factor	Product
$\tfrac{1}{4} \times \tfrac{1}{5} = \square$	$\tfrac{1}{4}$		$\tfrac{1}{20}$
$\tfrac{1}{3} \times \tfrac{1}{7} = \square$		$\tfrac{1}{7}$	
$\tfrac{2}{3} \times \tfrac{1}{5} = \square$			

(ii) Copy each of the following sentences by replacing the ○ with =, >, or < to make it true :

$3\frac{2}{5} \times 1\frac{1}{4} \bigcirc 2\frac{1}{4} \times 2\frac{4}{5}$
$2\frac{2}{3} \times 1\frac{7}{8} \bigcirc 2\frac{1}{3} \times 2\frac{1}{8}$

(iii) Rewrite each of the following equations replacing △ with +, −, or ×, so that each sentence is true :

$\frac{1}{2} \triangle \frac{1}{2} = 1$
$\frac{1}{2} \triangle \frac{1}{2} = 0$
$\frac{1}{2} \triangle \frac{1}{3} = \frac{1}{6}$
$\frac{1}{2} \triangle \frac{1}{3} = \frac{5}{6}$

(iv) A recipe requires $2\frac{3}{4}$ cups of water. If mother wants to make only half the quantity in the recipe, how much water will she require ?

(v) Circle any exercise below for which the product of the factors is less than the second factor. Draw a line under any exercise below for which the product of the factors is less than the first factor.

$\frac{3}{8} \times \frac{8}{9}$
$\frac{2}{3} \times 3$
$\frac{1}{3} \times \frac{11}{12}$

(vi) Copy and complete each sentence below, correctly, by writing >, <, or = in place of ○.

$\frac{1}{2} \times \frac{1}{4} \bigcirc 1 \qquad 4 \times \frac{3}{8} \bigcirc 1$
$\frac{2}{7} \times 5 \bigcirc 1 \qquad \frac{1}{7} \times 7 \bigcirc 1$

6. Fractions and the Properties of Numbers

The commutative property for multiplication has previously been applied to whole numbers. This property may now be applied to the multiplication of fractions. The identity property and the distributive property of multiplication over addition or subtraction should also be considered.

Commutative property of multiplication, e.g. $\frac{2}{3} \times \frac{1}{5} = \frac{1}{5} \times \frac{2}{3}$

Distributive property of multiplication over addition or subtraction, e.g. $\frac{3}{4} \times 6\frac{1}{2} = \square$

$$\begin{aligned} &\tfrac{3}{4} \times 6\tfrac{1}{2} \\ &= \tfrac{3}{4} \times (6 + \tfrac{1}{2}) \\ &= \tfrac{3}{4} \times 6 + \tfrac{3}{4} \times \tfrac{1}{2} \\ &= \tfrac{18}{4} + \tfrac{3}{8} \\ &= \tfrac{36}{8} + \tfrac{3}{8} \\ &= \tfrac{39}{8} = 4\tfrac{7}{8} \end{aligned}$$

or

$$\begin{aligned} &\tfrac{3}{4} \times 6\tfrac{1}{2} \\ &= \tfrac{3}{4} \times (8 - 1\tfrac{1}{2}) \\ &= \tfrac{3}{4} \times 8 - \tfrac{3}{4} \times 1\tfrac{1}{2} \\ &= 6 - \tfrac{3}{4} \times \tfrac{3}{2} \\ &= 6 - \tfrac{9}{8} \\ &= 6 - 1\tfrac{1}{8} \\ &= 5 - \tfrac{1}{8} = 4\tfrac{7}{8} \end{aligned}$$

Examples of written activities are presented below :

Commutative property of multiplication

(i) Use information given in the table to solve each of the following equations :

TABLE

$1\frac{3}{7} \times \frac{2}{5} = \frac{20}{35}$	$\frac{3}{7} \times 1\frac{2}{5} = \square$
$3\frac{7}{8} \times 2\frac{3}{4} = 10\frac{21}{32}$	$3\frac{5}{8} \times 2\frac{1}{4} = \square$
$\frac{3}{7} \times 1\frac{2}{5} = \frac{21}{35}$	$\frac{2}{5} \times 1\frac{3}{7} = \square$
$2\frac{1}{4} \times 3\frac{5}{8} = 8\frac{5}{32}$	$2\frac{3}{4} \times 3\frac{7}{8} = \square$

(ii) If you are given that $\frac{4}{7} \times \frac{9}{12} = \frac{3}{7}$, which of the following equations can you solve without further working ?

$\frac{4}{7} \times \frac{5}{8} = \square$ $\qquad$ $\frac{9}{12} \times \frac{3}{7} = \square$

$\frac{3}{7} \times \frac{4}{7} = \square$ $\qquad$ $\frac{9}{12} \times \frac{4}{7} = \square$

$\frac{3}{4} \times \frac{4}{7} = \square$

7. Shortening Computation

At this level the term "cancelling" should not be used. In Section F the child was encouraged to seek series of equal fractions. He saw : $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots$ as being different names or numerals for the same number. The child knows that if the numerator and the denominator of any fraction are multiplied or divided by the same number, an equal fraction is obtained.

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

$$\frac{6}{15} = \frac{6 \div 3}{15 \div 3} = \frac{2}{5}$$

Previous experience has also equipped the child with the knowledge that the number 1 can be renamed as $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}$, and so on. From this background of experience and knowledge, plus an examination of the basic properties discussed above, the child should develop methods which reduce the computational aspects of multiplication of fractions for him.

What children are really doing when they shorten their work is rearranging the factors in the numerator and the factors in the denominator in such a way that one of the fractions represents the number one. In doing this they are applying the commutative property of multiplication and the identity law.

$$\begin{aligned}\frac{5}{7}\times\frac{7}{9} &= \frac{5\times 7}{7\times 9}\\ &= \frac{5\times 7}{9\times 7}\quad \text{(Commutative property)}\\ &= \frac{5}{9}\times\frac{7}{7}\\ &= \frac{5}{9}\times 1\\ &= \frac{5}{9}\quad \text{(Identity property of 1)}\end{aligned}$$

$$\begin{aligned}\frac{6}{7}\times\frac{2}{3} &= \frac{6\times 2}{7\times 3} = \frac{2\times 3\times 2}{7\times 3}\\ &= \frac{2\times 2\times 3}{7\times 3}\\ &= \frac{4\times 3}{7\times 3} = \frac{4}{7}\times\frac{3}{3} = \frac{4}{7}\times 1\\ &= \frac{4}{7}\end{aligned}$$

C. DIVISION OF VULGAR FRACTIONS

1. In this section the child is introduced to the division of a fraction by a whole number and the division of a whole number by a fraction. From his experiences of division of whole numbers by fractions, and of fractions by whole numbers, the child should discover that—

(*a*) when the divisor decreases, the quotient increases :

(i) $8 \div \frac{1}{2} = 16$	(ii) $\frac{1}{2} \div 8 = \frac{1}{16}$
$8 \div \frac{1}{4} = 32$	$\frac{1}{2} \div 4 = \frac{1}{8}$
$8 \div \frac{1}{8} = 64$	$\frac{1}{2} \div 2 = \frac{1}{4}$

(*b*) if the divisor remains constant and the dividend decreases, the quotient decreases :

(i) $8 \div \frac{1}{2} = 16$	(ii) $\frac{1}{2} \div 2 = \frac{1}{4}$
$4 \div \frac{1}{2} = 8$	$\frac{1}{4} \div 2 = \frac{1}{8}$
$2 \div \frac{1}{2} = 4$	$\frac{1}{6} \div 2 = \frac{1}{12}$

2. Interrelationships between four operations.—The child has discovered and used the interrelation between the four operations when operating with whole numbers. Before this topic—division involving fractions—is introduced, division of whole numbers by whole numbers should be revised. In this review the following points should

be emphasised. First, that for the counting numbers, division may be understood as successive subtraction. Second, that division is the inverse or opposite of multiplication.

Utilisation of the interrelation of the processes is valuable. However, not all ideas of interrelationships with whole numbers can be expected to carry across into the wider system of fractions, for example, $4 \div \frac{1}{2} =$, can be solved by successive subtraction utilising the equation $4 - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = 0$, which can be rewritten as $4 - 8 \times \frac{1}{2} = 0$.

$\frac{3}{8} \div 3$ *cannot* be related to the notion of successive subtractions.

3. "Invert the divisor and multiply."—This mechanical procedure should not be introduced in developing the topic. The child may ultimately adopt this process, but he should do so as an outcome of his own experience and through his own generalisation from examples he has worked. Some children will need many varied experiences to make the generalisation, while others will require relatively few.

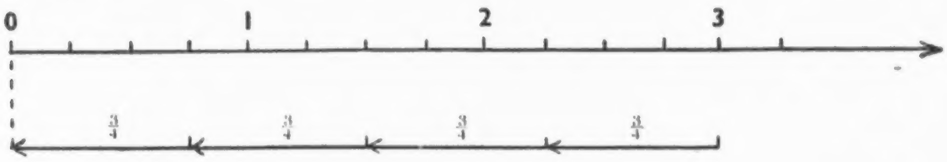
Development

(The following development does not necessarily present the only suitable sequence. Some steps may conveniently be reordered, or even excluded, depending upon the background experiences of the child.)

(*a*) *Division as Successive Subtraction*

It was through successive subtraction that the child was introduced to the computation procedure for division with whole numbers. Such an approach seems to be a satisfactory one with which to introduce division of whole numbers by fractions. Use can be made of the number line and rods to assist the development of the child's understanding in this matter.

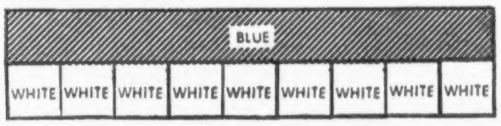
Example : $3 \div \frac{3}{4} = \square$



Example : $3 \div \frac{1}{3} = \square$

LIGHT-GREEN

is called "one".



When light-green is called one, the value of blue becomes three.
When light-green is called one, the value of white becomes one-third.
"How many thirds in three ?"

"There are nine thirds in three."

Use could also be made of a pegboard with interlocking pegs.

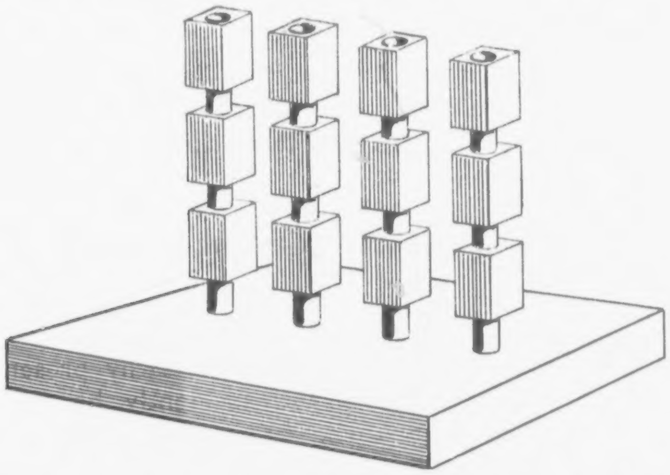
Example : $4 \div \frac{1}{3} = \square$

How many columns of pegs ? (4)

Each peg is what part of a column ? ($\frac{1}{3}$)

How many thirds in four columns ? (12)

To discover the answer to the last question, the child may physically remove, in turn, each peg from the board.



The computational procedure of subtraction can also be carried out.

Example : $4 \div \frac{2}{3} = \square$

$$\begin{array}{r|r|l} \frac{2}{3} & 4 & \\ & -\frac{2}{3} & 1 \\ \hline & 3\frac{1}{3} & \\ & -\frac{2}{3} & 1 \\ \hline & 2\frac{2}{3} & \\ & -\frac{2}{3} & 1 \\ \hline & 2 & \\ & -\frac{2}{3} & 1 \\ \hline & 1\frac{1}{3} & \\ & -\frac{2}{3} & 1 \\ \hline & \frac{2}{3} & \\ & -\frac{2}{3} & 1 \\ \hline & 0 & 6 \end{array}$$

$4 \div \frac{2}{3} = 6$

(b) *Division as the Inverse of Multiplication*

The child will recall that equations for whole numbers dealing with multiplication and division can be written in pairs such as $4 \times 3 = 12$; $12 \div 4 = 3$.

It is said that division is the opposite or inverse of multiplication. The child is now ready to ask whether a similar relationship for multiplication/division holds for fractions. A study of pattern will provide him with the answer. For example :

$$\begin{array}{rcl} 8 \times 1 = 8 & \qquad & 8 \div 1 = 8 \\ 16 \times \frac{1}{2} = 8 & & 8 \div \frac{1}{2} = 16 \\ 32 \times \frac{1}{4} = 8 & & 8 \div \frac{1}{4} = 32 \\ 64 \times \frac{1}{8} = 8 & & 8 \div \frac{1}{8} = 64 \end{array}$$

After studying a *number* of such pairs the child should discover that division as the inverse of multiplication also holds for fractions.

(c) *The Reciprocal Property of Rational Numbers*

Consider a pair of fractions such as $\frac{5}{3}$ and $\frac{3}{5}$:

$$\frac{5}{3} \times \frac{3}{5} = \frac{15}{15} = 1.$$

Two such fractions whose product is 1 are called *reciprocals* of each other.

Graded exercises of the following kinds are useful to establish the idea of reciprocal pairs :

(i) Find the products :

$\frac{5}{8} \times \frac{8}{5} = \square$ $\qquad$ $\cdot 1 \times 10 = \square$

$\frac{6}{1} \times \frac{1}{6} = \square$ $\qquad$ $1\frac{1}{4} \times \frac{4}{5} = \square$

$\frac{3}{4} \times \frac{4}{3} = \square$

(ii) Find the missing factor :

$\square \times \frac{1}{5} = 1$ $\qquad$ $6 \times \square = 1$

$\frac{4}{5} \times \square = 1$ $\qquad$ $\square \times 1\frac{1}{2} = 1$

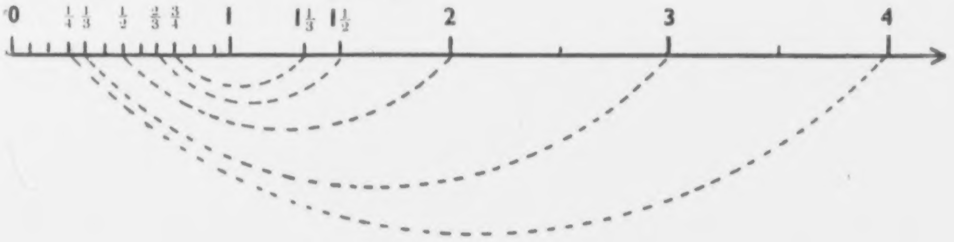
$\square \times \frac{7}{4} = 1$

(iii) Two-thirds is the reciprocal of $\square$.

(iv) Replace ○ by +, −, ×, or ÷ :

$$\frac{1001}{1000} \bigcirc \frac{1000}{1001} = 1$$

(v) Children might map the relationship between pairs of reciprocals on a number line.



It will be noticed that all reciprocals of numbers greater than one pair with numbers between 0 and 1.

1 is the reciprocal of itself.

0 has no reciprocal in the fractions because division by 0 is undefined—we cannot consider $\frac{x}{0}$ and consequently $\frac{0}{x} \times \frac{x}{0}$ is a meaningless expression.

(*d*) *Before proceeding further the following ideas should be reviewed* :

(i) The operation of division can be symbolised by fractional notation.

For example : $20 \div 4$ can be rewritten as $\frac{20}{4}$
$2 \div 5$ can be rewritten as $\frac{2}{5}$
and $5 \div \frac{3}{5}$ can be rewritten as $\frac{5}{\frac{3}{5}}$

(ii) When both the numerator *and* the denominator of a fraction are multiplied or divided by the same number, the quotient is unaffected.

$$\text{Thus : } \frac{1}{3} = \frac{1 \times 8}{3 \times 8} = \frac{1 \times 100}{3 \times 100} = \frac{1 \times 3\frac{1}{3}}{3 \times 3\frac{1}{3}}$$

$$\left(\frac{1}{3} = \frac{8}{24} = \frac{100}{300} = \frac{3\frac{1}{3}}{10}\right)$$

$$\frac{12}{24} = \frac{12 \div 2}{24 \div 2} = \frac{12 \div 3}{24 \div 3} = \frac{12 \times \frac{1}{4}}{24 \times \frac{1}{4}}$$

$$\left(\frac{12}{24} = \frac{6}{12} = \frac{4}{8} = \frac{3}{6}\right)$$

(iii) Division by 1 leaves the dividend unchanged

$$\frac{5}{1} = 5 ; \quad \frac{\frac{4}{5}}{1} = \tfrac{4}{5} ; \quad \frac{\frac{3}{4}}{\frac{5}{5}} = \frac{3}{4}$$

(*e*) *An understanding of division by a fraction comes to different children in different ways.* Two possible ways are as follows :

(i) Consider $7 \div \frac{2}{5} = \square$

$7 \div \frac{2}{5}$ may be rewritten as $\frac{7}{\frac{2}{5}}$ *or* $7 \div \frac{2}{5}$

$$\text{Then } \frac{7}{\frac{2}{5}} = \frac{7}{\frac{2}{5}} \times 1 = \frac{7}{\frac{2}{5}} \times \frac{\frac{5}{2}}{\frac{5}{2}}$$

$$\frac{7 \times \frac{5}{2}}{\frac{2}{5} \times \frac{5}{2}} = \frac{7 \times \frac{5}{2}}{1}$$

$$= 7 \times \tfrac{5}{2} = \tfrac{35}{2} = 17\tfrac{1}{2}$$

Hence

$$7 \div \tfrac{2}{5} = 7 \times \tfrac{5}{2} = 17\tfrac{1}{2}$$

$$= (7 \times \tfrac{5}{2}) \div (\tfrac{2}{5} \times \tfrac{5}{2})$$

$$= 7 \times \tfrac{5}{2} \div 1$$

$$= 7 \times \tfrac{5}{2} = \tfrac{35}{2} = 17\tfrac{1}{2}$$

Hence

$$7 \div \tfrac{2}{5} = 7 \times \tfrac{5}{2} = 17\tfrac{1}{2}$$

(ii) $7 \div \frac{2}{5} = \square$

Using the understanding of division as the inverse of multiplication,

$7 \div \frac{2}{5} = \square$ can be rewritten as $\frac{2}{5} \times \square = 7$

and $\frac{5}{2} \times \frac{2}{5} \times \square = \frac{5}{2} \times 7$

Hence $1 \times \square = \frac{5}{2} \times 7$

$\square = \frac{35}{2} = 17\frac{1}{2}$

Then $7 \div \frac{2}{5} = 7 \times \frac{5}{2} = 17\frac{1}{2}$.

Thus the original exercise, $7 \div \frac{2}{5}$, has been rewritten in each case as $7 \times \frac{5}{2}$ and completed. The child will have arrived at the same result (in a plausible way) as he would have by employing the rule " invert the divisor and multiply ". By following the above procedures, however, the rule itself is formulated as an outcome of understandings.

(*f*) *An understanding of division of a fraction by a whole number can come to different children in different ways.*

Experience with a variety of methods can also strengthen this understanding. Three possible ways are given below :

(i) Consider $\frac{3}{4} \div 5 = \square$

$$\frac{3}{4} \div 5 = \frac{\frac{3}{4}}{5}$$

$$= \frac{\frac{3}{4}}{5} \times 1$$

$$= \frac{\frac{3}{4}}{5} \times \frac{\frac{1}{5}}{\frac{1}{5}} \qquad \text{(Renaming 1)}$$

$$= \frac{\frac{3}{4} \times \frac{1}{5}}{5 \times \frac{1}{5}}$$

$$= \frac{\frac{3}{4} \times \frac{1}{5}}{1} \qquad \text{(Renaming } 5 \times \tfrac{1}{5} \text{ as 1)}$$

$$= \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$$

Thus $\frac{3}{4} \div 5 = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

(ii) $\frac{3}{4} \div 5 = \square$

$\frac{3}{4} \div 5$

$= \frac{15}{20} \div 5$ (15 twentieths $\div$ 5 = 3 twentieths
i.e. $\frac{15}{20} \div 5 = \frac{3}{20}$)

$= \frac{3}{20}$

(iii) $\frac{3}{4} \div 5 = \square$

$\frac{3}{4} \div 5 = \frac{3}{4} \div \frac{5}{1} = \square$

that is $5 \times \square = \frac{3}{4}$ (Multiplication as inverse of division)

that is $\frac{1}{5} \times \frac{5}{1} \times \square = \frac{1}{5} \times \frac{3}{4}$

Hence $1 \times \square = \frac{1}{5} \times \frac{3}{4} = \frac{3}{4} \times \frac{1}{5}$

and $\square = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

$\square = \frac{3}{20}$

Thus $\frac{3}{4} \div 5 = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$

D. ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS

The child has had experience in adding and subtracting decimals in Section G. His work is now extended to include hundredths. More than three addends may be used, but in subtraction only one operation should be taken, e.g. $56{\cdot}42 - 21{\cdot}71 = \square$.

The extension of this topic to include hundredths presents few new difficulties. Development and activity suggestions set out in the Guide to Section G (pages 66 to 69) for addition and subtraction may well be considered in extending the topic to hundredths. Review of Decimal Notation—Extension to Hundredths, Curriculum Guide, Section G (pages 17, 18), should also be made.

The child should widen his understandings of the basic properties of arithmetic to include the application of these properties to decimals.

1. Money and Decimals

Previously, up to and including Section G, when adding or subtracting money, the amounts have been considered as dollars and cents, not as parts of dollars. Understanding of dollars and cents is now used to develop a knowledge of decimal fractions.

(*a*) (i) $\$1 = \square$ cents

$1\text{c} = \$\,\square$

$\frac{1}{100}$ of $\$1 = \square\text{c}$

(ii) $\$0{\cdot}04 = \square$ cents

$6\text{c} = \frac{\square}{100}$ of $\$1$

$\$\square = 7\text{c}$

(iii) $\square$ ten cents $= \$1$

4 ten–cent coins $= \$\square$

(*b*) (i)
$$\begin{array}{r} \$1{\cdot}46 \\ +\quad \$3{\cdot}21 \\ \hline \\ \hline \end{array}$$

(ii)
$$\begin{array}{r} \$0{\cdot}46 \\ +\quad \$1{\cdot}07 \\ \hline \\ \hline \end{array}$$

(iii)
$$\begin{array}{r} \$5{\cdot}26 \\ -\quad \$2{\cdot}14 \\ \hline \\ \hline \end{array}$$

$$\begin{array}{r} 1{\cdot}46 \\ +\quad 3{\cdot}21 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 0{\cdot}46 \\ +\quad 1{\cdot}07 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 5{\cdot}26 \\ -\quad 2{\cdot}14 \\ \hline \\ \hline \end{array}$$

(iv) $$\begin{array}{r} \$3\cdot24 \\ -\quad \\ \$1\cdot41 \\ \hline \\ \hline \end{array}$$ (v) $$\begin{array}{r} \$\boxed{\quad\cdot\quad} \\ -\quad \\ \$\boxed{\quad\cdot\quad} \\ \hline \\ \hline \end{array}$$

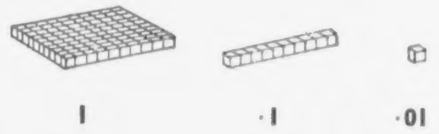
$$\begin{array}{r} 3\cdot24 \\ -\quad \\ 1\cdot41 \\ \hline \\ \hline \end{array} \qquad \begin{array}{r} 4\cdot02 \\ -\quad \\ 3\cdot14 \\ \hline \\ \hline \end{array}$$

In selecting examples attention should be given to regrouping, zeros, and difficulties involving blank spaces.

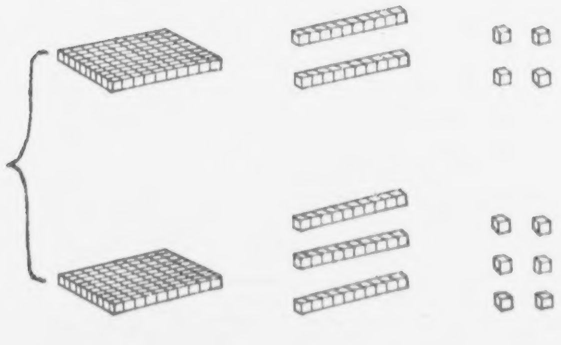
2. Structured Aids

The use of structured aids such as Dienes's Multibase Arithmetic Blocks and Cuisenaire rods in addition and subtraction of decimal fractions involving hundredths :

(i) *Dienes's Multibase Arithmetic Blocks*
Designate a " flat " as 1. The " long " then becomes ·1 and the " unit ", ·01.



Example
$$\begin{array}{r} 1\cdot24 \\ +\quad \\ 1\cdot36 \\ \hline \\ \hline \end{array}$$



(ii) *Cuisenaire Rods*
This aid can be used in a way similar to that outlined in (i) above. Orange would be called ·1. Other rods then become hundredths (white, ·01 ; yellow, ·05 ; red, ·02 ; and so on).

Once the pattern of addition and subtraction with whole numbers is seen by the child to extend to decimals, it is not expected that concentrated work using these structured aids need continue.

3. Restating Decimal Fractions in Vulgar Fraction Form

It is not expected that much attention need be paid to this procedure in order to develop the formal processes of addition and subtraction of decimal fractions. This idea, in the initial stages, may appeal to some.

$$\begin{aligned} & 0{\cdot}03 + 0{\cdot}18 \\ &= \tfrac{3}{100} + \tfrac{18}{100} \\ &= \tfrac{21}{100} \\ &= {\cdot}21 \end{aligned}$$

4. Numerical Examples

Numerical examples, similar to those undertaken with addition and subtraction of vulgar fractions, can be adapted to work with decimals. For example :

(*a*) Make each statement true by replacing the ○ with, >, < or = :

- (i) 29·58 + 30·71 ○ 113·42 — 53·23
- (ii) 986·42 — 439·14 ○ 794·02 — 247·08

(*b*) Solve the following :

- (i) 47 hundredths + 257 hundredths = □ hundredths
- (ii) $\frac{47}{100} + \frac{257}{100} = \square$
- (iii) ·47 + 2·57 = □
- (iv) 4·7 tenths + 3 hundredths = □ hundredths

(*c*) Solve the following :

- (i) 307 hundredths — 29 hundredths = □ hundredths
- (ii) $\frac{307}{100} - \frac{29}{100} = \square$
- (iii) $\triangle - {\cdot}29 = 2\frac{78}{100}$
- (iv) 2 — □ = 1·95
- (v) ·7 — ·38 = □

(*d*) Solve the following :

- (i) 214·32 + □ + 105·27 = 421·28
- (ii) 51·04 — □ = 42·09
- (iii) 591·17 — 30·27 = □
- (iv) 2 + ·04 + 38·9 = □

5. Commutative and Associative Properties of Addition with Decimals

Examples of suitable activities for this aspect of the topic are given in the discussion on fractions. (See pages 35–36 of this Guide.)

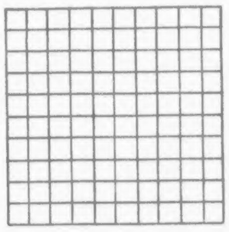
E. MULTIPLICATION OF DECIMAL FRACTIONS

It is at this stage that the child is introduced to multiplication of decimals by decimals. Prior to this he was multiplying decimals by whole numbers.

Difficulties in computation should be limited—products should be restricted to three decimal places and multipliers to two digits.

A teacher can approach this topic in numerous ways. The following are suggestions which can be varied according to the background and the maturity of the child. For some children it may be necessary to present a visual model of multiplying decimals by decimals. Other children may commence the topic by utilising a particular piece of mathematical understanding. While each child may favor one particular method, ultimately he should see and use a variety of approaches.

One approach to this topic is that which uses a visual model for the ideas presented. A diagram which could be made from $\frac{1}{4}$-inch graph paper is used. A square, showing a 10 by 10 grid, is used as a unit. Because of the child's background experiences with value relations, he should have no difficulty in establishing that each small square is one-hundredth of the complete unit.



Questions to help understanding might be :

"How many small squares are there in the unit square ?"

"One small square is what part of the unit square ?"

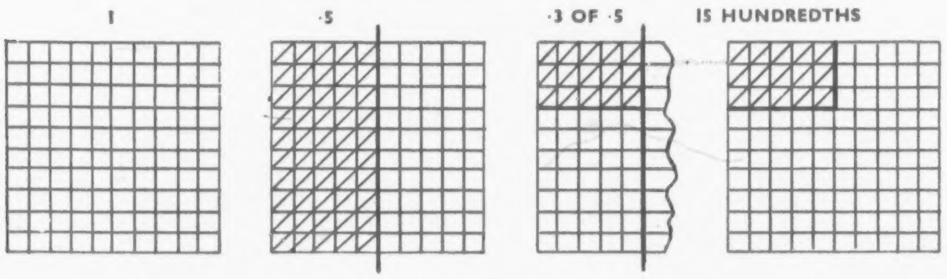
"Two small squares are what part of the unit square ?"

"How many hundredths make one unit square ?"

"If the large square is called 1, what is the value of four small squares ?"

With the example ·3 × ·5, the child first finds five-tenths of the unit and then three-tenths of five-tenths. As each square is one-hundredth of the whole, the child finds by counting that three-tenths of five-tenths is fifteen-hundredths.

(Here × is verbalised as of.)



An alternative approach utilises the child's background in vulgar fractions. He should be led to adapt his understanding of the multiplication of vulgar fractions to the multiplication of decimals.

Before children actually reach the stage of carrying out the operation of multiplication, they will need to be given practice in renaming vulgar fractions as decimals, and decimals as vulgar fractions.

$$\cdot 4 = \frac{\square}{10}$$

$$\tfrac{14}{100} = \cdot\square$$

$$2\cdot 4 = \frac{\square}{10} = \frac{24}{\square}$$

$$5\cdot 7 = \triangle + \frac{7}{\square} = \frac{\triangledown}{100}$$

Consider $\cdot 3 \times \cdot 5 = \square$

Rewrite $\cdot 3 \times \cdot 5$ as $\tfrac{3}{10} \times \tfrac{5}{10}$

$$\tfrac{3}{10} \times \tfrac{5}{10} = \tfrac{15}{100} = \cdot 15$$

Hence $\cdot 3 \times \cdot 5 = \cdot 15$

Similarly,

$$\begin{array}{llllll}
\text{(i)} & \cdot 15 \times \cdot 6 & \text{(ii)} & \cdot 01 \times 15 & \text{(iii)} & 2\cdot 8 \times 3\cdot 1 \\
 & = \tfrac{15}{100} \times \tfrac{6}{10} & & = \tfrac{1}{100} \times \tfrac{15}{1} & & = \tfrac{28}{10} \times \tfrac{31}{10} \\
 & = \tfrac{90}{1000} & & = \tfrac{15}{100} & & = \tfrac{868}{100} \\
 & = \cdot 090 & & = \cdot 15 & & = 8\cdot 68
\end{array}$$

Once the child has mastered the mechanics of the operation, his attention should be drawn to the relationship that exists between the product and two factors. This can be done by means of a table as presented below. From an examination of numerous examples, the child may discover that when two decimal fractions are multiplied there are as many decimal places in the product as there are in the sum of the decimal places in the factors. (This section of the work will probably be taken late in Section H or early in Section I.) For example, $\cdot 7 \times \cdot 8 = \cdot 56$.

Product of tenths by tenths is hundredths.

Problem	Decimal places in first factor	Decimal places in second factor	Product	Decimal places in product
·7 × ·8	1	1	·56	2
·24 × 2	☐	0	·48	☐
·6 × 4	☐	☐	☐	1

Place the decimal points correctly in the underlined numerals :

$\cdot 3 \times \cdot 7 = \underline{21}$ $\qquad$ $\underline{7} \times \cdot 3 = 2 \cdot 1$

A variety of methods may be used to carry out and to record exercises. A child might verbalise in vulgar fraction form while he records in decimal form. Another might record in vulgar fractions after having rewritten the exercise in this form, and convert back to decimal form when recording his answer. It is important that the equation be written in full at the conclusion of the exercise. For example :

(i) $\cdot 3 \times \cdot 8 = \square$

$$\begin{aligned} &\cdot 3 \times \cdot 8 \\ &= (\cdot 1 \times 3) \times (\cdot 1 \times 8) \\ &= \cdot 1 \times \cdot 1 \times 3 \times 8 \\ &= \cdot 01 \times 24 \\ &= \cdot 24 \end{aligned}$$

$$\boxed{\cdot 3 \times \cdot 8 = \cdot 24}$$

(This procedure might profitably be discussed in relation to basic properties and place value.) The child will verbalise as : " One-tenth multiplied by one-tenth equals one-hundredth. One-hundredth multiplied by twenty-four equals 24 hundredths."

Another child might work and record in vulgar fractions.

(ii) $1 \cdot 3 \times 2 \cdot 8 = \square$

$$\begin{aligned} &1\tfrac{3}{10} \times 2\tfrac{8}{10} \\ &= (1 + \tfrac{3}{10}) \times (2 + \tfrac{8}{10}) \\ &= 1 \times (2 + \tfrac{8}{10}) + \tfrac{3}{10} \times (2 + \tfrac{8}{10}) \\ &= 2 + \tfrac{8}{10} + \tfrac{6}{10} + \tfrac{24}{100} \\ &= 2 + \tfrac{14}{10} + \tfrac{24}{100} \\ &= 3 + \tfrac{4}{10} + \tfrac{24}{100} \\ &= 3 + \tfrac{6}{10} + \tfrac{4}{100} \\ &= 3 \cdot 64 \end{aligned}$$

$$\boxed{1 \cdot 3 \times 2 \cdot 8 = 3 \cdot 64}$$

or $1 \cdot 3 \times 2 \cdot 8$

$$\begin{aligned} &= (1 + \cdot 3) \times (2 + \cdot 8) \\ &= 1 \times (2 + \cdot 8) + \cdot 3 \times (2 + \cdot 8) \\ &= 2 + \cdot 8 + \cdot 6 + \cdot 24 \\ &= 3 \cdot 64 \end{aligned}$$

$$\begin{array}{r} 2 \cdot 00 \\ \cdot 24 \\ \cdot 6 \\ \cdot 8 \\ \hline 3 \cdot 64 \\ \hline \end{array}$$

$$\boxed{1 \cdot 3 \times 2 \cdot 8 = 3 \cdot 64}$$

In the above examples, renaming techniques and the basic properties of number have been freely used. The child should be as familiar with the uses of these properties with both vulgar and decimal fractions as he was in the context of whole numbers.

Once a child has seen the plausibility of the generalisation $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ in terms of decimals, he may adopt a whole number approach when solving incomplete equations. Such a procedure may assist accuracy in computation. He may ignore the decimal points in the factors and treat each as a whole number. To determine the correct placement of the decimal point, a procedure of rounding off and estimation is used. Consider $2 \cdot 3 \times \cdot 9$. By ignoring the decimal points the child obtains the answer 207. The estimation of 2 × 1 = 2 indicates the plausibility of placing the decimal point between the two and the zero to give an answer of 2·07.

Consider the example : 53·2 × 7·8 = □

$$\begin{aligned} &53{\cdot}2 \times 7{\cdot}8 \\ &= \tfrac{532}{10} \times \tfrac{78}{10} \\ &= \tfrac{41496}{100} \\ &= 414{\cdot}96 \end{aligned}$$

$$\begin{array}{r} 532 \\ 78 \\ \hline 4{,}256 \\ 37{,}240 \\ \hline 41{,}496 \\ \hline \end{array}$$

53·2 × 7·8 = □

(50 × 8 = 400, hence the answer to 53·2 × 7·8 is approximately 400)

532 × 78 = 41,496

then 53·2 × 7·8 = 414·96

$$= \boxed{53{\cdot}2 \times 7{\cdot}8 = 414{\cdot}96}$$

The child should use the rounding off procedure to check his answers in examples such as that above. In the example, answers such as 4·1496, 4,149·6, 41,496 or ·41496 would obviously be unreasonable.

F. DIVISION OF DECIMAL FRACTIONS

In this section the child is introduced to the division of a decimal by a whole number and to the division of a whole number by a decimal. Difficulties in computation are restricted. Divisions should be exact (that is, they should not involve remainders) and quotients should be limited to two decimal places. Divisors should be either tenths or whole numbers.

A Study of Pattern

A useful way of developing division of decimals is through a study of pattern. Consider the following :

(i) 200 ÷ 100 = 2
200 ÷ 10 = 20
200 ÷ 1 = 200
200 ÷ ·1 = □

(ii) 200 ÷ 10 = 20
100 ÷ 10 = 10
50 ÷ 10 = 5
25 ÷ 10 = □
12·5 ÷ 10 = □

(iii) 500 ÷ 10 = 50
50 ÷ 10 = 5
5 ÷ 10 = □
□ ÷ 10 = △

(iv) 300 ÷ 3 = □
30 ÷ 3 = □
3 ÷ 3 = □
·3 ÷ 3 = □
·03 ÷ 3 = □

(v) 500 ÷ 100 = □
50 ÷ 100 = □
□ ÷ 100 = ·05

(vi) 10 ÷ 50 = ·2
10 ÷ 5 = □
10 ÷ ·5 = □

(vii) $16 \div 4 = \square$
$8 \div 2 = \square$
$4 \div 1 = \square$
$2 \div \square = 4$

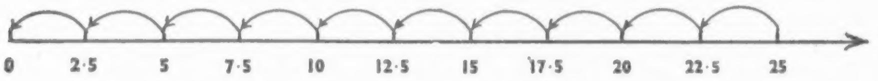
(viii) $1 \div 1 = 10$
$2 \div \cdot 1 = \square$
$4 \div \cdot 1 = \square$
$8 \div \cdot 1 = \square$

(ix) $1 \div \cdot 1 = 10$
$1 \div \cdot 2 = 5$
$1 \div \cdot 4 = \square$
$1 \div \cdot 8 = \square$

By the use of a number line or by computation using the understanding of division as the inverse of multiplication, the child can check the answers he has discovered through a consideration of pattern.

The Number Line (the ruler can be very useful here)

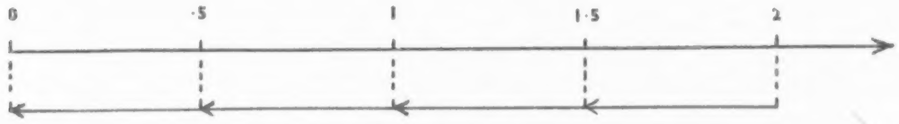
(i)



$25 \div 10 = 2{\cdot}5$ $[2{\cdot}5 \times 10 = 25]$

(25 is partitioned into 10 equal parts.)

(ii)



$2 \div \cdot 5 = 4$ (·5 is subtracted **four** times.)

The ideas of partitioning into equal parts and successive subtraction can be used with structured aids such as Cuisenaire rods, or with money.

At a later stage (and many children might operate at this level without reference to concrete aids) the child might visualise the situation and verbalise to obtain the required answer. For example :

(i) $2{\cdot}5 \div 5 = \square$. " If I divide 2·5 inches into 5 equal parts, how long is each ? "

(ii) $1 \div \cdot 2 = \square$. " How many times can I take ·2 inches from 1 inch ? "

Clearly, the child must distinguish when it is appropriate to invoke successive subtraction and when partitioning is required.

Prior to this section, the child has worked with examples where the divisor has always been smaller than the dividend. Now he meets examples in which the divisor will sometimes be larger than the dividend. In such situations successive subtraction is inappropriate. However, the idea of partitioning (when the divisor is a whole number) can be used. Partitioning can be illustrated by the child cutting wool or strips of paper.

Take a strip of paper and cut the following lengths : 1·6 in. and 2·8 in. Cut the 1·6 in. piece into two equal parts and record the information asked for in the following table. Cut the 2·8 in. piece into four equal parts and record as before.

Length of strip	Number of pieces	Length of one part

The child can then be encouraged to relate the two situations above to an appropriate equation.

$1{\cdot}6 \div 2 = \square$	$2{\cdot}8 \div 4 = \square$
$1{\cdot}6 \times 2 = \square$	$2{\cdot}8 + 4 = \square$
$1{\cdot}6 + 2 = \square$	$2{\cdot}8 \times 4 = \square$

Multiplication as the Inverse of Division

In order to establish this relationship with decimals, examples such as the following might be used :

(i) Find the missing numbers in the pairs of equations below :

${\cdot}3 \times 5 = \square$ $1{\cdot}5 \div {\cdot}3 = \triangle$	${\cdot}5 \times 6 = \square$ $3{\cdot}0 \div {\cdot}5 = \triangle$	${\cdot}4 \times 8 = \square$ $3{\cdot}2 \div {\cdot}4 = \triangle$
$5{\cdot}7 \times 3 = \square$ $\square \div 5{\cdot}7 = \triangle$	$3{\cdot}9 \times 4 = \square$ $\square \div 4 = 3{\cdot}9$	$4{\cdot}8 \times 4 = \square$ $\square \div 4 = \triangle$

(ii) Solve each equation in the left-hand column and write the related division equation, without further calculation, in the right-hand column :

$5{\cdot}6 \times 4 = \square$	$22{\cdot}4 \div 4 = 5{\cdot}6$ $\triangle \div \square = 4$
$7{\cdot}6 \times 8 = \square$	$\triangle \div \square = \Diamond$ $\diamond \div \nabla = \bigcirc$

It is important for the child to come ultimately to an understanding of division as the inverse (or opposite) of multiplication. This understanding of division is appropriate for all kinds of numbers, including counting numbers, whole numbers, vulgar fractions, and decimal fractions. Nevertheless, ideas of division related to partition, quotition, and successive subtraction should not be neglected since these often provide the cue or clue to understanding of verbal problems.

Division of Decimal Fractions as an Extension of Division Using Whole Numbers

The child should be led to see the computational procedure for division involving decimal fractions as an extension of division using whole numbers. A review of the skills involved in division with whole numbers is important before extending the process to work with decimals. Consider, for example, the child's ability to estimate partial quotients.

7321 ÷ 7 = □	4956 ÷ 5 = □	3204 ÷ 9 = □
7 × 1 = 7 7 × 10 = 70 7 × 100 = 700 7 × 1000 = □	9 × 5 = 45 90 × 5 = 450 900 × 5 = □	□ × 9 = 27 30 × 9 = 270 300 × 9 = □

4624 ÷ 6 = □	5246 ÷ 4 = □	4008 ÷ 8 = □
46 ÷ 6 ⩾ □ 460 ÷ 6 ⩾ □ 4600 ÷ 6 ⩾ □	52 ÷ 4 ⩾ □ 520 ÷ 4 ⩾ □ 5200 ÷ 4 ⩾ □	40 ÷ 8 ⩾ □ 400 ÷ 8 ⩾ □ 4000 ÷ 8 ⩾ □

Note : (*a*) ⩾ means "is greater than or equal to".

(*b*) In the above examples, □ stands for the largest whole number for which the sentence is true.

A review of place value can be undertaken through examples such as :

(i) Rename 65·1 as—

☐ tens ☐ ones ☐ tenths
☐ ones ☐ tenths
☐ tenths
☐ hundredths

(ii) Rename 27·91 as—

……………hundredths
……………tenths ……………hundredths
……………ones ……………tenths ……………hundredths
……………tens ……………ones ……………tenths ……………hundredths

The child should be encouraged to use the following steps when solving division equations involving decimals :

(*a*) Estimate the quotient ;
(*b*) work the example as though only whole numbers were used ;
(*c*) place the decimal point according to the estimate ;
(*d*) check the answer gained through use of the inverse operation —multiplication.

Developmental Exercises

(i) Place the decimal points correctly in the quotients of the following examples :

8·61 ÷ 7 = 123
53·6 ÷ 6 = 89
4·20 ÷ 5 = 84

(ii) Complete the following table :

	→	→	→
14 ÷ 2 = ☐	138 ÷ 2 = ☐	13·8 ÷ 2 = ☐	☐ × 2 = 13·8
21 ÷ 7 = ☐	203 ÷ 7 = ☐	20·3 ÷ 7 = ☐	☐ × 7 = 20·3
	→	→	→

An alternative approach is to use the renaming techniques that the child has developed.

3·48 ÷ 4 = ☐

3·48 can be thought of as 348 hundredths, and the exercise as 348 hundredths ÷ 4.

	4	348	
348 hundredths ÷ 4		320	80
= 87 hundredths		28	
		28	7
3·48 ÷ 4 = ·87		0	87

The child should be encouraged to check his answer by using the inverse operation—multiplication : ·87 × 4 = ☐

PERCENTAGE

The child will be concerned with percentage only as an alternative notational form for certain vulgar and decimal fractions. He should be able to associate fractions with denominators of 100 and percentages. Fractions which can be renamed as fractions with denominators of 100 —hundredths, fiftieths, twentieths, tenths, quarters, and halves—can be written as percentages. For example,

$$\tfrac{3}{100} = 3\%;$$
$$\tfrac{3}{20} = \tfrac{15}{100} = 15\%;$$
$$\tfrac{3}{4} = \tfrac{75}{100} = 75\%;$$
$$\cdot 7 = \tfrac{7}{10} = \tfrac{70}{100} = 70\%.$$

Computational exercises are not suggested. Project work might be undertaken and the child encouraged to discover where percentages are commonly used.

SECTION I

RATIO

A ratio can be considered simply as a comparison between two numbers or two quantities of the same kind. A ratio can be recorded in fraction form. The ratio p is to q (q not equal to 0) can be written $p : q$ or as the quotient $\dfrac{p}{q}$.

The idea of ratio is involved in all measurement. When two quantities are compared as in length measurement, a chosen length (usually called a unit) is compared with a length to be measured. For example, the ratio 2 : 5 (2 is to 5) is another way of recording the fraction $\tfrac{2}{5}$. If this ratio expresses the relationship between the lengths of two pieces of wood, we can say that the longer piece is five times as long as half of the shorter length. If a sum of money is to be shared so that two boys share it in the ratio of 2 : 5, then one boy receives $\tfrac{2}{7}$ and the other boy $\tfrac{5}{7}$ of the total amount.

RATE

Whereas *ratio* is concerned with like quantities *rate* is concerned with the expression of the relationship that might exist between unlike quantities, such as miles and hours, articles and money, bags of wheat and acres, and so on.

Thus a motor-car which travels 50 miles (at a steady speed) in two hours is travelling at a rate of $\frac{50}{2}$ miles per hour. Similarly, if four dozen eggs cost \$2·80 they are sold at the rate of $\frac{2 \cdot 80}{4}$ dollars per dozen.

ADDING AND SUBTRACTING VULGAR FRACTIONS

Skills and understandings achieved in Section H should be maintained. Computational examples such as the following might be used :

(i) $\frac{3}{5} + \frac{1}{6} + 2\frac{2}{3} = \square$

(ii) $3\frac{2}{7} + 5\frac{1}{3} + 13 = \square$

(iii) $3\frac{7}{8} - 1\frac{2}{3} - 1\frac{5}{12} = \square$

This variation (No. iii above) can be investigated in terms of the basic properties, associativity and commutativity. For example :

Replace the $\square$ with $=$ or $\neq$ in the following :

$(\frac{3}{5} + \frac{1}{6}) + 2\frac{2}{3} \quad \square \quad \frac{3}{5} + (\frac{1}{6} + 2\frac{2}{3})$
$(3\frac{7}{8} - 1\frac{2}{3}) - 1\frac{5}{12} \quad \square \quad 3\frac{7}{8} - (1\frac{2}{3} - 1\frac{5}{12})$
$3\frac{7}{8} - 1\frac{2}{3} - 1\frac{5}{12} \quad \square \quad 3\frac{7}{8} - 1\frac{5}{12} - 1\frac{2}{3}$
$3\frac{7}{8} - 1\frac{2}{3} - 1\frac{5}{12} \quad \square \quad 1\frac{2}{3} - 3\frac{7}{8} - 1\frac{5}{12}$

MULTIPLICATION OF FRACTIONS

Examples can be extended to include three factors. The basic properties—commutative, associative, distributive, identity, and inverse—should be considered and used in simplifying fractions. For example :

(*a*) $\frac{1}{2} \times (\frac{3}{4} \times 4) = \square$; $(\frac{1}{2} \times \frac{3}{4}) \times 4 = \square$; $(4 \times \frac{1}{2}) \times \frac{3}{4} = \square$

(*b*) $3\frac{1}{3} \times 3 \times \frac{1}{5} = \square$; $3\frac{1}{3} \times \frac{1}{5} \times 3 = \square$; $3\frac{1}{3} \times (3 \times \frac{1}{5}) = \square$
(The commutative and the associative properties are frequently used in conjunction in the one example. Consider $3\frac{1}{3} \times \frac{1}{5} \times 3 = \square$. This can be rewritten $\frac{1}{5} \times 3 \times 3\frac{1}{3} = \square$, using the commutative or "any order" property and regrouping for working as $\frac{1}{5} \times (3 \times 3\frac{1}{3}) = \square$, using the associative property.)

(*c*) Choose (without carrying out any working) the correct answer from the answers supplied :
$2\frac{1}{2} \times 3\frac{1}{3} \times 3 = \square$
(i) 25, (ii) $\frac{18}{6}$, (iii) 150, (iv) $18\frac{1}{6}$

(*d*) $\frac{15}{16} \times 1\frac{3}{5} \times \frac{1}{3}$

This example might be worked in a variety of ways :

(i) $$\frac{15}{16} \times \frac{8}{5} \times \frac{1}{3}$$

$$= \frac{15}{16} \times \left(\frac{8}{5} \times \frac{1}{3}\right)$$

$$= \frac{15}{16} \times \frac{8}{15}$$

$$= \frac{15 \times 8}{15 \times 16}$$

$$= \frac{15}{15} \times \frac{8}{16}$$

$$= \frac{8}{16} = \frac{1}{2}$$

(ii) $$\left(\frac{15}{16} \times \frac{8}{5}\right) \times \frac{1}{3}$$

$$= \frac{15 \times 8}{5 \times 16} \times \frac{1}{3}$$

$$= \frac{5 \times 3 \times 8}{5 \times 2 \times 8} \times \frac{1}{3}$$

$$= \frac{5 \times 8}{5 \times 8} \times \frac{3}{2} \times \frac{1}{3}$$

$$= \frac{3 \times 1}{3 \times 2} = \frac{1}{2}$$

(iii) $$\frac{15}{16} \times \frac{1}{3} \times \left(1 + \frac{3}{5}\right)$$

$$= \frac{5 \times 3}{16 \times 3} \times \left(1 + \frac{3}{5}\right)$$

$$= \frac{5}{16} \times 1 + \frac{5}{16} \times \frac{3}{5}$$

$$= \frac{5}{16} + \frac{3 \times 5}{16 \times 5}$$

$$= \frac{5}{16} + \frac{3}{16} = \frac{8}{16} = \frac{1}{2}$$

In examples such as the one above, the child should be encouraged to explain what has been done, step by step. In his own working the child will, in time, omit to record many of the steps set out above.

DIVISION OF FRACTIONS

The child should continue to have experience with the types of examples he has worked in Section H. In addition, he might now carry out exercises involving division of a fraction by a fraction.

Many ways of carrying out this computation are available to the child. Each involves understandings he has already achieved. A number of worked examples are set out below. The examples are developed step by step—it is expected that the child will, of his own accord, soon omit to record many of them. It is possible that he may generalise to the extent of applying the rule " invert the divisor and multiply ".

Examples :

(*a*) (i) $\frac{3}{4} \div \frac{1}{4} = \square$ (3 quarters ÷ 1 quarter = $\square$ "How many times can I take 1 quarter from 3 quarters ?" Answer : 3.)

$\frac{3}{4} \div \frac{1}{4} = 3$

(ii) $\frac{3}{4} \div \frac{1}{12} = \square$ ($\frac{3}{4}$ can be renamed as $\frac{9}{12}$; 9 twelfths ÷ 1 twelfth = $\square$)

$\frac{9}{12} \div \frac{1}{12} = 9$

$\frac{3}{4} \div \frac{1}{12} = 9$

(iii) $\frac{3}{5} \div \frac{3}{4} = \square$ ($\frac{3}{5} = \frac{12}{20}$, $\frac{3}{4} = \frac{15}{20}$ 12 twentieths ÷ 15 twentieths = $\square$)

Clearly, the ideas of partitioning and of successive subtraction are inappropriate in solving this problem. The method used in (i) and (ii) above will not satisfy (iii).

(*b*) (i) $\frac{3}{4} \div \frac{1}{4} = \square$

$$= \frac{3}{4}\Big/\frac{1}{4}$$

$$= \frac{\frac{3}{4} \times \frac{4}{1}}{\frac{1}{4} \times \frac{4}{1}}$$

$$= \frac{3}{4} \times \frac{4}{1}$$

$$= \frac{3 \times 4}{4 \times 1}$$

$$= \frac{3 \times 4}{1 \times 4}$$

$$= \frac{3}{1} \times \frac{4}{4}$$

$$= 3 .$$

$$\frac{3}{4} \div \frac{1}{4} = 3$$

(ii) $\frac{3}{4} \div \frac{1}{12} = \square$

$$= \frac{\frac{3}{4}}{\frac{1}{12}}$$

$$= \frac{\frac{3}{4} \times \frac{12}{1}}{\frac{1}{12} \times \frac{12}{1}}$$

$$= \frac{3}{4} \times \frac{12}{1}$$

$$= \frac{3 \times 12}{4 \times 1}$$

$$= \frac{3 \times 3 \times 4}{1 \times 4}$$

$$= \frac{9}{1} \times \frac{4}{4} = 9$$

$$\frac{3}{4} \div \frac{1}{12} = 9$$

(iii) $\frac{3}{5} \div \frac{3}{4} = \square$

$$= \frac{\frac{3}{5}}{\frac{3}{4}}$$

$$= \frac{\frac{3}{5} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}}$$

$$= \frac{3 \times 4}{5 \times 3}$$

$$= \frac{3}{3} \times \frac{4}{5}$$

$$= \frac{4}{5}$$

$$\frac{3}{5} \div \frac{3}{4} = \frac{4}{5}$$

(c) Applying the inverse operation—multiplication :

(i) If $\frac{3}{4} \div \frac{1}{4} = \square$

then $\frac{1}{4} \times \square = \frac{3}{4}$

$$\left(\frac{4}{1} \times \frac{1}{4}\right) \times \square = \frac{4}{1} \times \frac{3}{4}$$

$$1 \times \square = \frac{4 \times 3}{4 \times 1}$$

$$\square = 3$$

(ii) If $\frac{3}{4} \div \frac{1}{12} = \square$

then $\square \times \frac{1}{12} = \frac{3}{4}$

and $\square \times \frac{1}{12} \times \frac{12}{1} = \frac{3}{4} \times \frac{12}{1}$

$$\square \times 1 = \frac{3 \times 3 \times 4}{4 \times 1}$$

$$= \frac{3 \times 3}{1} \times \frac{4}{4}$$

$$= \frac{3 \times 3}{1} \times 1 = 9$$

(iii) $$\begin{array}{r} \frac{3}{5} \div \frac{3}{4} = \square \\ \square \times \frac{3}{4} = \frac{3}{5} \\ \frac{3}{5} \div \frac{3}{4} = \frac{4}{5} \end{array} \qquad \left[\begin{array}{l} \square \times \frac{3}{4} \times \frac{4}{3} = \frac{3}{5} \times \frac{4}{3} \\ = \dfrac{3 \times 4}{5 \times 3} = \dfrac{3 \times 4}{3 \times 5} = \dfrac{4}{5} \end{array} \right]$$

It will be noticed that in each of the above cases, (*a*), (*b*), and (*c*), the division problem is restated as a multiplication exercise.

ADDITION AND SUBTRACTION OF DECIMALS

Skills and understandings gained in earlier sections should be maintained. Exercises might include thousandths, and more than three addends might be considered. Exercises involving both addition and subtraction of decimals could be used.

Examples involving fractions of quantities, exactly expressible in thousandths, can be used (e.g. 3 gal. 1 pint = 3·125 gal. ; 13 yd. 9 in. = 13·25 yd. ; 7 cwt. 42 lb. = 7·375 cwt. ; 75c = $0·75).

Where computation is involved, the use of decimal notation can provide an alternative means of solution and may, in many cases, provide a more economical and satisfying means. (These remarks can also be applied to multiplication and, in some cases, division. Care should be taken to ensure that exercises do not become unnecessarily complex or unrealistic.)

Suggested Examples :

(*a*) 3·08 + 0·253 + 253 − ·308 = □

(*b*) 17·5 + □ − 7·5 = 21·001

(*c*) Choose an answer from those suggested below which you think is the best estimate for the answer to : 1·304 + 0·99 + ·099 − 2·001

(i) 1·3 ; (ii) ·3 ; (iii) 0

(*d*) $17·25 + $0·05 + 35c − $\boxed{\$\quad}$ = $17·10

(e) In a walkathon those taking part were sponsored at the rate of 4c a mile. Six girls took part and the following distances were checked : Louise (4·2 miles), Linda (5·25 miles), Frances (5·65 miles), Susan (6·1 miles), Wendy (6·8 miles), and Robyn (8·3 miles). How much money did they raise ?

(*f*) Find the missing terms (indicated by frames) in the sequence : 1·123 ; 2·246 ; 3·369 ; 5·615 ; 8·984 ; $\boxed{14\cdot\quad}$; $\boxed{\quad\quad}$.

MULTIPLICATION OF DECIMALS

In Section H the child had arrived at the stage where he could employ a formal algorithm to assist him in the multiplication of decimals. Ideas and skills developed then should be maintained. Computations can be increased in difficulty and might include examples where products have three decimal places. The child should be encouraged to check his answers for plausibility by rounding off factors as suggested in Section H.

Suggested Examples :

(*a*) 11·1 × 1·11 = □

(*b*) ·01 × 0·1 = □

(*c*) Which of the possible answers given is the best approximation for 31·09 × 3·9 ?
(i) 128 ; (ii) 96 ; (iii) 93 ; (iv) 121.

(*d*) A man calculated that it cost him 1·75c per mile for tyre wear (·35c for each of 5 tyres), 2·3c per mile for petrol, 1·1c per mile for registration and insurance, and 1·85c per mile for repairs and maintenance other than tyres. How much did he pay for one year's running of his car if he travelled 10,326 miles ?

DIVISION OF DECIMALS

When the child has confidence in solving division equations he can investigate the pattern that exists between the number of decimal places in the dividend, the divisor, and the quotient.

The number of decimal places in the quotient is the same as the number of decimal places in the dividend *less* the number of places in the divisor. For example :

15·6 ÷ ·4 = 39   76·5 ÷ ·3 = 255
1·56 ÷ ·4 = 3·9   7·65 ÷ ·3 = 25·5
·156 ÷ ·4 = ·39   ·765 ÷ ·3 = 2·55

Use the pattern seen in these examples to choose the correct quotient in each of the following :

95·4 ÷ ·6 = □   (159, 1·59, or 15·9)
13·456 ÷ 1·6 = □   (8·41, 84·1, or 841)

Pupils can now be asked to solve examples such as :

17·80 ÷ ·5 = □
2·87 ÷ ·7 = □

Again, the child should be encouraged to estimate an answer, make the necessary calculations, place the decimal point by using the pattern discussed above, and finally check the answer by multiplication.

The traditional method of making the divisor a whole number could now be considered. First, the following ideas should be revised :

(*a*) The form of division notation can be changed from 342 ÷ 3 to $\frac{342}{3}$ without affecting the quotient.

(*b*) To change a decimal fraction to a whole number it is necessary to multiply it by 10 or a power of 10 (that is by 10, by 100, or by 1000, and so on).
5·92 × 100 = 592
16·7 × 10 = 167

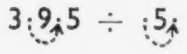
(*c*) A number multiplied by 1 remains unchanged : ½ × 1 = ½ ; 3·4 × 1 = 3·4 (the identity property of multiplication).

(*d*) There are many names for 1, e.g. $\frac{10}{10}, \frac{1\cdot5}{1\cdot5}, \frac{100}{100}, \frac{1}{1}$

Using these ideas the exercise 3·95 ÷ ·5 can be rewritten as 39·5 ÷ 5.

$$
\begin{aligned}
& 3\cdot95 \div \cdot5 & \\
&= \frac{3\cdot95}{\cdot5} & \text{(Changing notation)} \\
&= \frac{3\cdot95}{\cdot5} \times 1 & \text{(Identity property)} \\
&= \frac{3\cdot95}{\cdot5} \times \frac{10}{10} & \text{(Renaming 1)} \\
&= \frac{3\cdot95 \times 10}{\cdot5 \times 10} & \\
&= \frac{39\cdot5}{5} & \\
&= 39\cdot5 \div 5 &
\end{aligned}
$$

When the child understands this sequence he can use the traditional procedure of changing the position of the decimal point in both the divisor and the dividend in order to obtain a divisor that is a whole number. The quotient will remain unchanged. For example :

3·95 ÷ ·5 = ☐ 
39·5 ÷ 5 = 7·9
Hence 3·95 ÷ ·5 = 7·9

Examples :

Rewrite each of the following so that the divisor becomes a whole number, while the quotient remains unchanged.

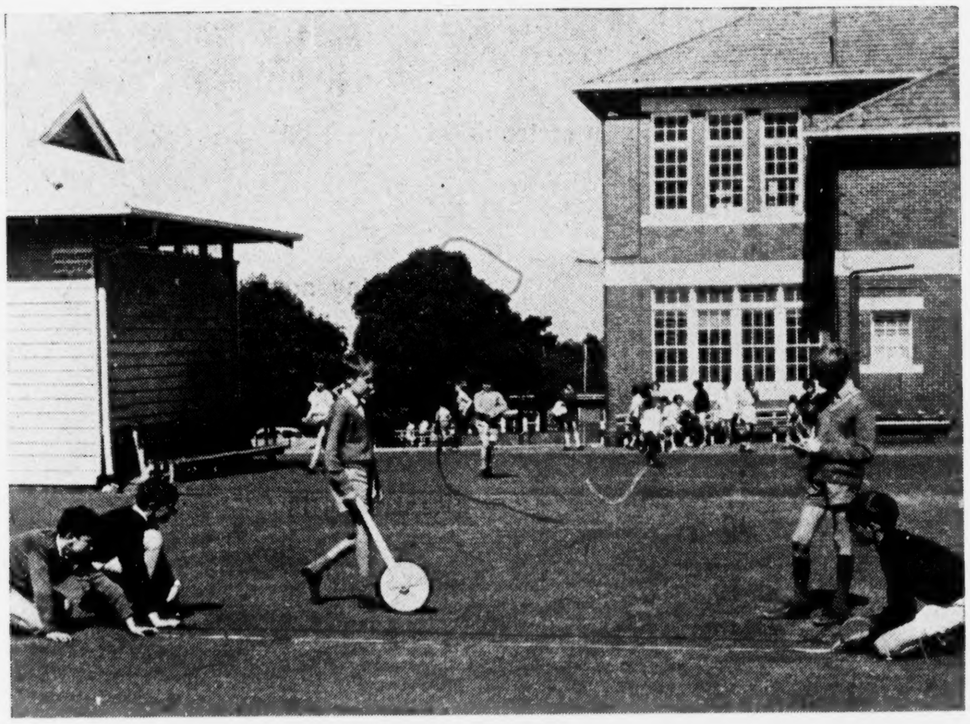
9·3 ÷ ·3 = 31
0·93 ÷ ·3 = 3·1
·1 ÷ ·1 = 1
10 ÷ ·1 = 100
·121 ÷ 1·1 = ·11

PERCENTAGE

Exploratory work in the use of percentage might be continued and extended.

Problems involving simple computation exercises can be introduced. Exercises such as the following might be used :

6% of 200 = ☐
3% of 60 = ☐
15% of $3 = ☐

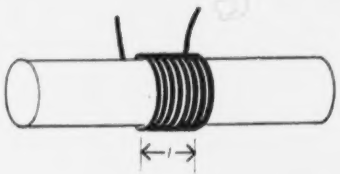


LENGTH

SECTION H

In Section H, and subsequently in Section I, the child should continue to develop his ability to estimate length. Whenever measurement is made it should be preceded by an estimate. The child should, by now, be surprised if there is a significant difference between his estimate of a distance and its measurement. The practice of "estimate *then* measure" should be frequently emphasised. Distances of 1 mile, ½ mile, ¼ mile, furlong, chain, ½ chain, yards, and feet should be meaningful for the child. The precision obtained in his estimations will, of course, be related to the magnitude of the length measured. The child's estimate of a mile may be as much as ten chains too large or too small ; that of a yard, a few inches either way. Children should still be encouraged to utilise personal, informal units such as paces or hand spans in making estimations.

Measurement activities may arise from problem situations. For example, a child may want to know the diameter of a wire. It may be discovered through discussion that, by coiling the wire around a stick, a measure can be obtained for a number of diameters. Perhaps eight or more turns may be made. The length of one diameter can be found by division.



Using a similar principle, the thickness of a page can be found. Distances that are not easily measured by using a ruler or a tape (perhaps due to considerable length, or to frequent turns, or both) can be measured by using a map wheel or a trundle wheel. (The advantages in using a trundle wheel as against a measuring stick should be considered. The measurement activity will suggest the better instrument when the advantages and the disadvantages of each instrument are known.)

Children should become familiar with the technique for measurement with a chain tape. In Section I, instruments such as callipers may be investigated. The degree of manipulative skill with these instruments will, of course, vary with individuals.

The child should, in Section H, have some understanding of precision measurement. He should appreciate the fact that there is a need for greater precision in particular situations and that in some others rough approximations only are possible. For example, a road distance from Melbourne to Mildura will not be accurately stated in yards, or even in chains ; a carpenter may need to measure with an accuracy of $\frac{1}{16}$ of an inch only while a mechanical engineer working on a motor may need to measure with an accuracy of ·001 of an inch.

Rounding off to given degrees of accuracy is an important skill that children in both Section H and Section I should continue to develop. Rounding off can and should be linked with pure number, especially in the division algorithm (see Curriculum Guide, Section G, page 50, and Sections H and I, pages 28 and 29) and in providing rough checks for answers in all process work. The child should constantly check his calculated answers in terms of their reasonableness, considering the data from which he is working.

The child should meet problems involving the direct measurement of perimeter. Although individual children will reduce actual measurement to the minimum necessary for them to calculate perimeter, it is not the objective of this section to formalise rules for such calculation. Practical devices may involve the use of string, the wheel, dividers, the edge of a sheet of paper, or a tape measure. Other means will doubtless suggest themselves to children.

The child should understand and use correct techniques for ruling lines of specified length. In Section H, accuracy should be to the nearest $\frac{1}{8}$ in. (that is, a maximum error of $\frac{1}{16}$ in. is allowed). The child should, however, attempt to attain the maximum accuracy he is able to achieve. Dividers are useful instruments in this work, both to mark the limits of lines to be ruled and to check the lengths of ruled lines.

The Furlong

The furlong is introduced to the child in Section H. A discussion which brings out how the need for this unit arose, and how it was established as a "furrow-long", can be interesting and instructive.

Attention might be drawn to the everyday use of the furlong in foot-running and horse-racing.

The child should have first-hand experience with the distance. He should step out a measured furlong, that is, find out how many of his own paces are needed to cover the distance. He should time himself in walking and in running one furlong. The child should measure a furlong in chains. Opportunities should be made for estimating a furlong and then measuring the estimated length in chains.

As an outcome of the child's knowledge that 10 chains = 1 furlong and that 80 chains = 1 mile, he will be able to establish by renaming and substitution the relationships : 220 yards = 1 furlong, and 8 furlongs = 1 mile.

ACTIVITIES INVOLVING COMPUTATION

Reduction

Reduction exercises should be restricted to two units. For example,

feet and inches to inches,
chains to furlongs and chains,
furlongs to yards.

Opportunities exist in these renaming exercises for direct checks by measurement in the required units.

Activities Leading to Formal Processes

These activities will, as was the case in Pure Number, Sections F and G, involve the application of the basic properties of arithmetic to length measures. Although the sequences set out below show a formal process as the final stage in development, it is not intended to suggest that the formal process is the computational form to be adopted in all exercises. On the contrary, the child should be encouraged to examine each problem with a view to choosing the most appropriate method at his disposal for its solution.

Addition

(*a*) Exercises involving feet and inches. For example :

$1 \text{ ft. } 8 \text{ in.} + 1 \text{ ft. } 11 \text{ in.} + 4 \text{ in.} = \square \text{ ft. } \square \text{ in.}$
$(1 \text{ ft.} + 8 \text{ in.}) + (1 \text{ ft.} + 11 \text{ in.}) + 4 \text{ in.}$
$= 1 \text{ ft.} + 1 \text{ ft.} + 8 \text{ in.} + 11 \text{ in.} + 4 \text{ in.}$ (Commutative property)
$= (1 + 1) \text{ ft.} + (8 + 11 + 4) \text{ in.}$
$= 2 \text{ ft.} + 23 \text{ in.}$
$= 2 \text{ ft.} + (1 \text{ ft.} + 11 \text{ in.})$ (Renaming)
$= (2 \text{ ft.} + 1 \text{ ft.}) + 11 \text{ in.}$ (Associative property)
$= 3 \text{ ft.} + 11 \text{ in.}$
$1 \text{ ft. } 8 \text{ in.} + 1 \text{ ft. } 11 \text{ in.} + 4 \text{ in.} = 3 \text{ ft. } 11 \text{ in.}$

Alternatively,

$$\begin{aligned}
&1\text{ ft. }8\text{ in.} + 1\text{ ft. }11\text{ in.} + 4\text{ in.}\\
&= (1\text{ ft. }8\text{ in.} + 4\text{ in.}) + 1\text{ ft. }11\text{ in.}\\
&= 2\text{ ft.} + 1\text{ ft. }11\text{ in.}\\
&= 3\text{ ft. }11\text{ in.}
\end{aligned}$$

(*b*) Addition involving yards and feet.

(*c*) Addition involving chains and yards.

(*d*) Addition involving furlongs and chains.

(*e*) Addition involving furlongs and miles.

(*f*) Addition involving chains and miles.

The development of a formal process for the addition of length may well follow lines similar to those set out for addition with whole numbers (see Curriculum Guide, Section F, pages 82–88). Most children will move very rapidly through the various stages of development and may even move directly to one of the final stages as set out below. This progress will depend largely on the child's understanding of addition with whole numbers and on his ability to generalise his knowledge of basic properties with whole numbers to quantities.

Example :

3 ft. 7 in.	3 ft. 7 in.	3 ft. 7 in.
5 ft. 9 in.	5 ft. 9 in.	5 ft. 9 in.
8 ft. 11 in.	8 ft. 11 in.	8 ft. 11 in.
16 ft. 27 in.	16 ft. 27 in.	18 ft. 3 in.
16 ft. + (2 ft. + 3 in.)	18 ft. 3 in.	
(16 + 2) ft. 3 in.		
18 ft. 3 in.		

(Note.—Detailed setting out may not be necessary for many children. The detail suggested may be of thought sequence rather than of recording.)

Subtraction

Development may well be graded as suggested for addition. The first stage should not involve subtraction in which bridging units occur. Early examples should be of a type easily checked by direct measurement. In later exercises, where larger lengths are involved, a number line can be used as a visual model for the situation.

A suitable gradation of exercises is seen in the following list of examples :

(i) What is the difference in length between 10 in. and 7 in.?

(ii) Subtract a length of 7 in. from 1 ft.

(iii) How much should I add to 7 in. to have $9\frac{1}{2}$ in.?

(iv) Subtract $7\frac{1}{2}$ in. from 10 in.

(v) 1 ft. 4 in. — 3 in.

(vi) What length must I add to $3\frac{1}{2}$ in. to have a length of 1 ft. 5 in.?

(vii) Subtract 1 ft. from 1 yard 2 ft.

(viii) What is the difference in length between 2 yards and 2 feet?

(ix) 19 yd. — 7 yd. = ☐ yd.

(x) What length do I need to add to 17 yd. to obtain 2 chains 10 yards?

(xi) Subtract 7 yd. from 1 ch. 2 yd.

(xii) 11 yd. 2 ft. — 7 yd. 1 ft.

(xiii) 11 yd. 1 ft. — 7 yd. 2 ft.

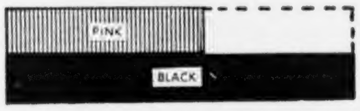
Many of these examples can be restated (and quantities in them renamed) to facilitate their solution in a variety of ways.

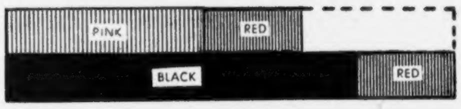
One important understanding which most children at this level should have gained is that when equals are added to unequals the difference remains constant. It is on this principle that the equal additions method of subtraction is based. The idea can be extended further and we may say: "When equals are added to or subtracted from unequals the difference is unchanged." Thus, using rods, the child discovers that—

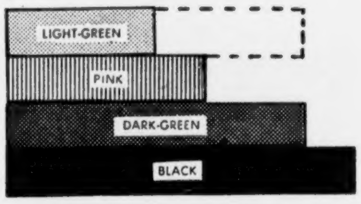
the difference between black and pink is equal to a light-green rod;

the difference between (black + red) and (pink + red) is still equal to a light-green rod; and

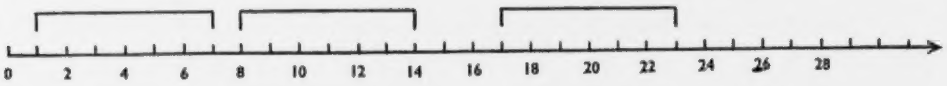
the difference between (black — white) and (pink — white) is also equal to a light-green rod.







The number line is a useful aid in this situation.



The difference between 8 and 14 is seen to be 6 units;

the difference between (8 + 9) and (14 + 9) is still seen to be 6 units; and

the difference between (8 — 7) and (14 — 7) is 6 units.

Using this principle in the examples given above, it is seen that many examples can be restated in more convenient terms. For example, in Exercise (x) and Exercise (xiii) above :

(x) 2 chains 10 yards — 17 yards can be restated as—
(2 chains 10 yards + 5 yards) — (17 yards + 5 yards)
= 2 ch. 15 yd. — 1 chain
= 1 ch. 15 yd.
2 chains 10 yards — 17 yards = 1 chain 15 yards

(xiii) 11 yd. 1 ft. — 7 yd. 2 ft. can be restated as—
11 yd. 2 ft. — 8 yd.
= 3 yd. 2 ft.
11 yd. 1 ft. — 7 yd. 2 ft. = 3 yd. 2 ft.

In some examples it may be better to carry out the subtraction of the larger unit first. For example :

17 ch. 17 yd. — 13 ch. 19 yd.
= 4 ch. 17 yd. — 19 yd. (an example of the "constant difference" principle)
= 4 ch. — 2 yd.
= 3 ch. 20 yd.
17 ch. 17 yd. — 13 ch. 19 yd. = 3 ch. 20 yd.

Using the equal additions method :

17 ch. 17 yd. — 13 ch. 19 yd.

$$\begin{array}{ll} & 39 \\ 17 \text{ ch.} & \cancel{17} \text{ yd.} \\ \cancel{13} \text{ ch.} & 19 \text{ yd.} \\ 14 & \\ \hline 3 \text{ ch.} & 20 \text{ yd.} \\ \hline \end{array}$$

17 ch. 17 yd. — 13 ch. 19 yd. = 3 ch. 20 yd.

Using the decomposition (renaming) method :

$$\begin{array}{ll} 16 & 39 \\ \cancel{17} \text{ ch.} & \cancel{17} \text{ yd.} \\ 13 \text{ ch.} & 19 \text{ yd.} \\ \hline 3 \text{ ch.} & 20 \text{ yd.} \\ \hline \end{array}$$

17 ch. 17 yd. — 13 ch. 19 yd. = 3 ch. 20 yd.

(No one method of subtraction should be generally regarded as superior. The particular problem should influence the method chosen.)

Multiplication

The child should be quite familiar with the property of distribution of multiplication over addition with respect to whole numbers. The teacher should now provide activities that will enable the child to apply meaningfully the distributive principles to quantities.

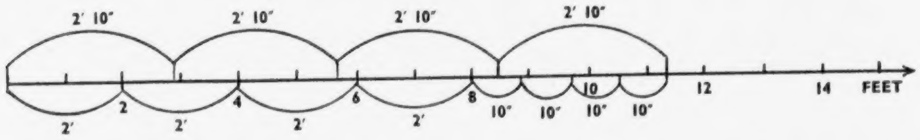
The child might be asked to take a string, say, 2 ft. 10 inches long, and to mark an interval equal to four lengths of the string along a line on the floor. He then cuts the string into two pieces, one piece two feet long and the other piece ten inches long. Placing the pieces along the measured interval the child finds that :

2 ft. 10 in. × 4 = 2 ft. × 4 + 10 in. × 4.

He can also discover, by taking four strings each 3 feet long and cutting 2 inches from each of the four, that :

3 ft. × 4 − 2 in. × 4 = 2 ft. 10 in. × 4

A number line is a very useful aid to illustrate this distributive property.



The child should be encouraged to rename the multiplicand in other ways, and then to check the results of multiplying each combination of quantities with the original interval.

Thus he might find :

2 ft. 10 in. × 4 = 11 ft. 4 in.
(2 ft. × 4) + (10 in. × 4) = 11 ft. 4 in.
(3 ft. − 2 in.) × 4 = 11 ft. 4 in.
(1 ft. + 1 ft. + 10 in.) × 4 = 11 ft. 4 in.
(2½ ft. × 4) + (4 in. × 4) = 11 ft. 4 in.

In a formal process, renaming is usually in terms of the units themselves. Thus 16 yd. 2 ft. is simply considered as 16 yd. + 2 ft. The multiplication is then carried out, using the distributive principle as follows :

(i) 16 yd. 2 ft. × 7
= (16 yd. + 2 ft.) × 7
= 16 yd. × 7 + 2 ft. × 7
= (16 × 7) yd. + (2 × 7) ft.
= 112 yd. + 14 ft.
= 112 yd. + 4 yd. + 2 ft.
= 116 yd. 2 ft.

(ii) 16 yd. + 2 ft.
× 7

112 yd. + 14 ft.
112 yd. + (4 yd. + 2 ft.)
116 yd. 2 ft.

Until the child is confident in multiplication of compound quantities, care must be taken to ensure that renaming in the final stages is not unnecessarily complex. For example, the following exercise involves such a difficulty :

3 chains 18 yards × 12
= 36 chains + 216 yards
= 36 chains + 9 chains + 18 yards
= 45 ch. 18 yd.

The child should be encouraged to estimate answers to problems. He should check his answers by estimates he has made. For the example above he might say : "Three chains eighteen yards is about four chains. Twelve times four is forty-eight—48 chains." His answer to the calculation is almost 46 chains, and so no gross error is made.

Division

Division involving length can have one of two distinct meanings. The child should consider the problem in hand and decide whether he wants to find—

(i) the number of equal lengths that can be obtained from a given length, or

(ii) the length of each of a number of equal lengths formed from a given length. (How many? (i) or How much? (ii))

The answer to the first question must be a counting number— 1, 2, 3, . . . The answer to the second is a quantity, it may involve fractions and will involve a unit of some kind—inches, feet, or some other unit. Division where the answer is to be a counting number is sometimes called *quotition* division ; where the answer is a quantity we have what is referred to as *partition* division.

For example :

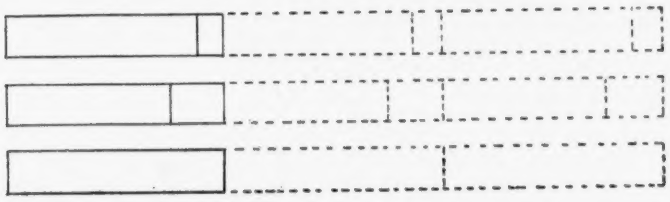
(i) How many pieces, each seven inches long, can I obtain from a piece of wire that is 22 inches long ? (Answer : 3.)

(ii) A piece of wire, 22 inches long, is to be divided into three equal pieces. What is the length of each piece ? (Answer : $7\frac{1}{3}$ inches.)

The child is familiar with the application of the distributive property of division over addition with the numbers of arithmetic. He must now widen his understanding of this property to include the distribution of division over quantities expressed in two units. Having made a similar extension in multiplication, he should speedily apply his knowledge to this new situation. Examples such as the following will assist :

> Take three paper strips, each 12 inches long. Divide the first one into three equal pieces. Cut the second twelve-inch strip into two pieces, one 9 inches long and the other 3 inches ; cut each piece into three equal parts. Cut the third twelve-inch strip into two at a point of your own choice ; divide each piece into three equal parts.

Take the first 4-inch piece. Above it place one of each of the equal pieces from the second strip. Do similarly for the third strip. What have you discovered ?



Recording each situation with an equation we have :

12 in. ÷ 3 = 4 in.

(9 in. + 3 in.) ÷ 3 = 3 in. + 1 in. = 4 in.

possibly ($10\frac{1}{2}$ in. + $1\frac{1}{2}$ in.) ÷ 3 = $3\frac{1}{2}$ in. + $\frac{1}{2}$ in. = 4 in.

Exercises should, at first, be simple enough to allow for direct checks. For example :

2 ft. 10 in. ÷ 4 = ☐

$$\begin{aligned} 2\text{ ft.} \div 4 + 10\text{ in.} \div 4 &= 6\text{ in.} + 2\tfrac{1}{2}\text{ in.} \\ &= 8\tfrac{1}{2}\text{ in.} \end{aligned}$$

This answer may be checked by multiplication—

$8\frac{1}{2}$ in. × 4 = (32 + 2) in. = 34 in. = 2 ft. 10 in.,

or by recourse to a 36-inch ruler where the length can be stepped off four times.

Later examples may be of the following type :

17 ch. 16 yd. ÷ 6 = ☐

One form of solution may be :

$$\begin{aligned} 17\text{ ch. } 16\text{ yd.} \div 6 &= (18\text{ ch.} - 6\text{ yd.}) \div 6 \\ &= (18 \div 6)\text{ ch.} - (6 \div 6)\text{ yd.} \\ &= 3\text{ ch.} - 1\text{ yd.} \\ &= 2\text{ ch. } 21\text{ yd.} \end{aligned}$$

A formal process for partition division may be set out as follows :

27 m. 15 ch. ÷ 5 = ☐

$$\begin{array}{r|r|l} 5 & 27\text{ m. } 15\text{ ch.} & \\ & 25\text{ m. } \ \ 0\text{ ch.} & 5\text{ miles} \\ \cline{2-2} & 2\text{ m. } 15\text{ ch.} & \\ & (160 + 15)\ \text{ch.} & \\ & 175\text{ ch.} & \\ & 150\text{ ch.} & 30\text{ ch.} \\ \cline{2-2} & 25\text{ ch.} & \\ & 25\text{ ch.} & 5\text{ ch.} \\ \cline{2-2} & 0 & 5\text{ m. } 35\text{ ch.} \end{array}$$

Answer :

27 m. 15 ch. ÷ 5 = 5 m. 35 ch.

Alternatively :

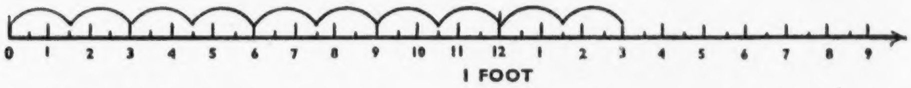
$$\begin{aligned} &27\text{ m. } 15\text{ ch.} \div 5 \\ &= (25\text{ m.} + 2\text{ m.} + 15\text{ ch.}) \div 5 \\ &= (25\text{ m.} + 160\text{ ch.} + 15\text{ ch.}) \div 5 \\ &= (25\text{ m.} + 175\text{ ch.}) \div 5 \\ &= 5\text{ m.} + 35\text{ ch.} \end{aligned}$$

Answer :

27 m. 15 ch. ÷ 5 = 5 m. 35 ch.

When the answer to a division problem is a number and not a quantity, a quite different approach may be required. A number line can assist in gaining understanding of the problem. Consider the example :

> How many labels, each $1\frac{1}{2}$ inches long, can be cut from a tape 1 foot 3 inches long ?



Answer : 10.

Doubling and halving techniques can be used meaningfully. For instance, consider the problem :

> A wheel has a perimeter of 4 feet. How many turns will the wheel make for a journey of 1 mile ?

Each revolution covers 4 feet, and the total distance is 5,280 feet. The child might ask : " How many times can I subtract 4 feet from the distance to be covered ? " He can restate the problem by asking : " How many lengths, each 4 feet, must I take to reach 5,280 feet ? "

Rename 5,280 ft. as (4,000 + 1,000 + 200 + 80) ft.

Now 4,000 ft. will be covered in 1,000 turns
1,000 ft. will be covered in 250 turns
200 ft. will be covered in 50 turns
80 ft. will be covered in 20 turns

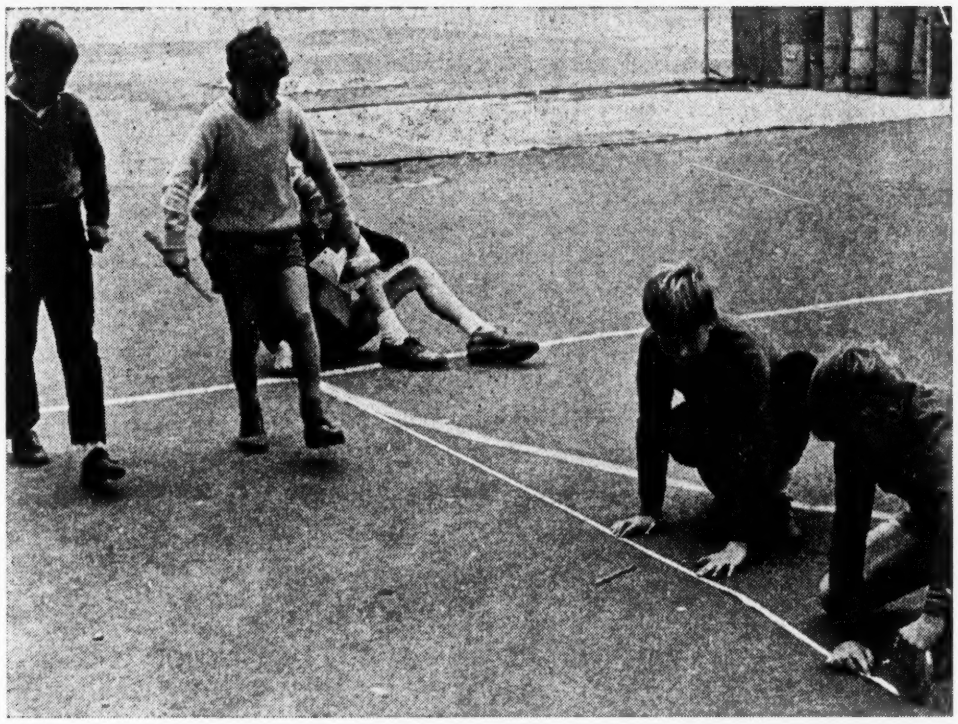
5,280 ft. 1,320 turns

Using doubling techniques :

1 turn covers 4 ft.
2 turns cover 8 ft.
4 turns cover 16 ft.
8 turns cover 32 ft.
800 turns cover 3,200 ft. } 4,800 ft. } 5,200 ft.
400 turns cover 1,600 ft.
100 turns cover 400 ft.
16 turns cover 64 ft.
4 turns cover 16 ft.

1,320 5,280 ft.

At this stage each problem should be considered in its own right and the most appropriate method (to the child) should be used to solve it. Eventually, with the aid of well-chosen questions, the child will recognise that problems involving quotition division reduce to processes in pure number. He should also be strongly aware of the need to express the dividend and the divisor in like units. These understandings are more likely to be developed if the child is confronted by many simple, realistic problems which are capable of direct practical solution than if he is presented with complex, meaningless exercises.



SECTION I

As the child proceeds with measurement topics he should become increasingly aware of the arbitrary nature of the units employed. This awareness can be strengthened by introducing the child to units of the metric system—the metre and the centimetre.

The child should carry out estimation and measurement activities using centimetres and metres. The relationship 100 centimetres = 1 metre may be discovered by direct measurement. The significance of the name "centimetre" could then be discussed. Measures may be written as, for example,

3·05 m. ; 305 cm. ; 3 m. 5 cm.

An investigation into the introduction of the system in France would seem most appropriate. The introduction of decimal currency to Australia serves as a useful parallel.

The advantages of the two systems of measurement, British and metric, should be considered. The decimal-based metric system facilitates computation. The British system allows for a much greater range of aliquot parts.

Some investigation into the incompatibility of the two measures will prove interesting and instructive. No whole number of units of one system corresponds to any whole number of units of the other. Equivalents can only be approximate (for example, 1 inch $\approx$ 2·54 cm.). This lack of exact equivalence makes the interchange of machinery parts (British and metric measures) impossible in most cases where fine precision is important. In Australia, a Senate committee has recently investigated educational, manufacturing, and other outcomes associated with a proposed change over to a metric system of measures. It would prove a useful and instructive activity, at Section I level, to investigate the costs associated with a change to a metric system of measures by any one industry. Consider, for example, the Victorian Railways. It should be noted that a change to metric units has occurred in some industries ; for example, the pharmaceutical industry.

Approximate equivalents can be discovered by children. Given a rod, one metre long, the child can discover by measurement that 1 m. ≈ 39 in. and that 1 in. ≈ 2·5 cm. Closer approximations can be obtained by equating lengths of, say, 10 in. and 25·4 cm. which leads to the approximation 1 in. ≈ 2·54 cm. An interesting approximation for athletics is that 100 m. is about 110 yd.

Perimeter determination should include both regular and irregular shapes. Children should be encouraged to make their own generalisations, for example, (i) " the circumference of a circle is about three times the diameter measurement ", or (ii) " the length of one side of a triangle is never greater than the sum of the lengths of the other two sides ". Such generalisations should be the outcome of direct, personal experiences with measurement. As has been the case in earlier sections, it is important that estimation should regularly precede measurement. Practice too should be continued in rounding off.

The child should be encouraged to make his own decisions as to the appropriate unit to which rounding off should take place. Centimetres and metres can now be used in estimation, measurement, and rounding off activities.

ACTIVITIES INVOLVING CALCULATION

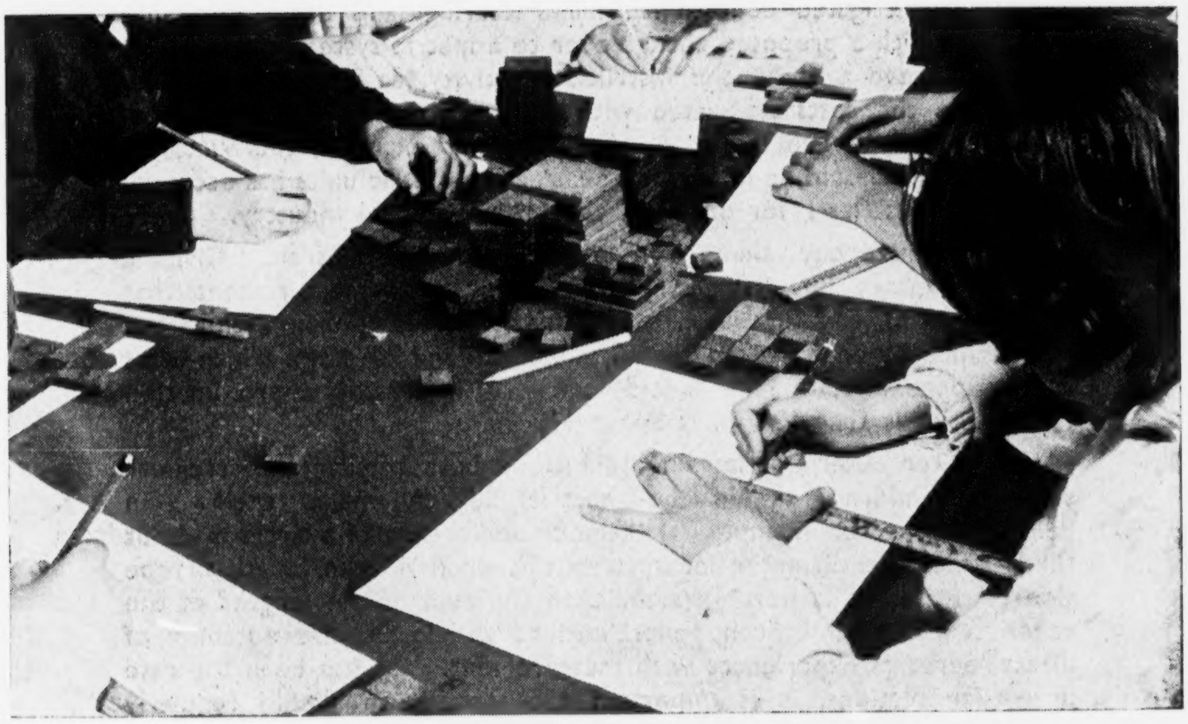
Relationships within the Tables

Relationships within the tables should be well understood and frequently used in calculations. The complexity of the relationships should be governed by the child's understanding of fractions at Section H level. Problems involving these relationships should be employed frequently and should, in general, be restricted to two units.

Formal Processes

Understanding of, and skill in, using formal processes with length measurements as developed in Section H should be maintained and further extended to include metres and centimetres. Process work should frequently result from problems taken from everyday life. Such problems need not be unduly complex and they should not, in general, involve more than two units.

Although it is necessary for the child to master formal processes providing a standard procedure, it is important that the use of other techniques for solution of equations be encouraged. A flexible approach to the solving of problems is an important objective of primary school mathematics.



AREA

SECTION H

The child, in Section G and earlier sections, has developed ideas about surfaces and area along the following lines :

(i) a surface has two dimensions ;

(ii) two or more regions can be compared in area ;

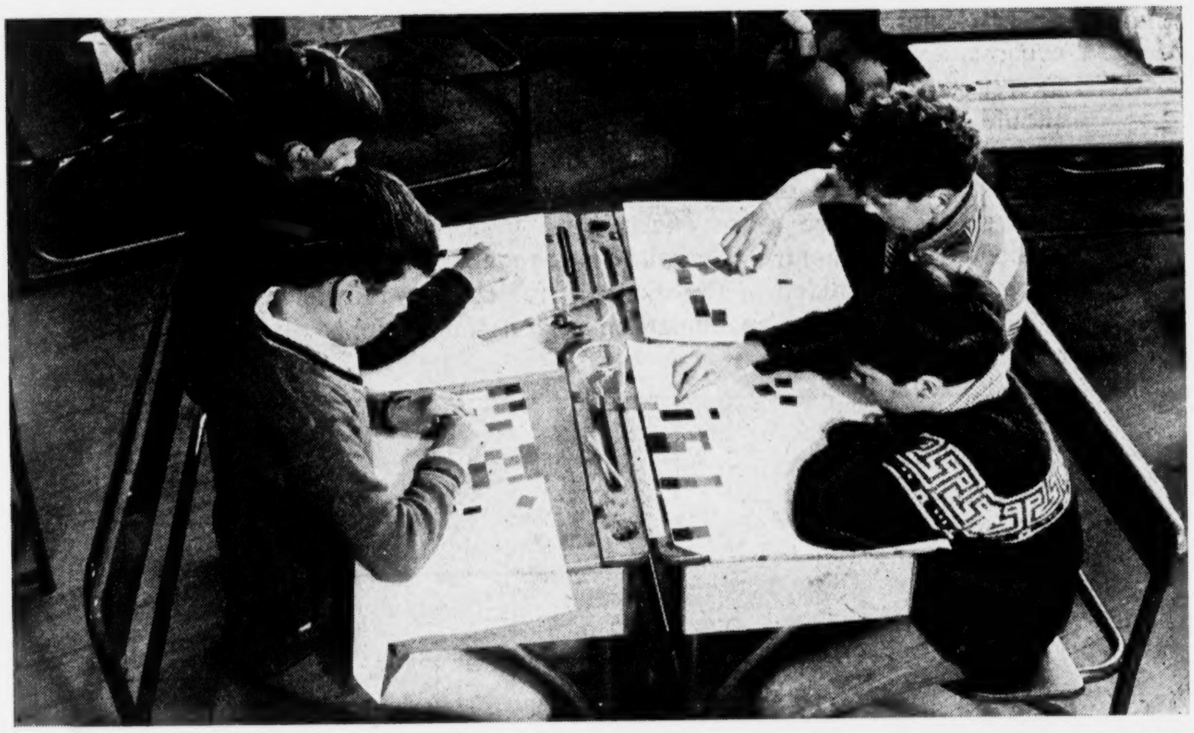
(iii) different shapes can have the same area ;

(iv) the unit for measuring regions of surface is itself a unit region ;

(v) some shapes are appropriate for measuring given regions, others are not.

A formal unit, as such, has not yet been introduced to the child. In some topics (length, weight, capacity) he has met and used formal units, and he should now have some appreciation of the usefulness of standardised units. It is an objective in Section H to introduce the square inch, the square foot, and the square yard.

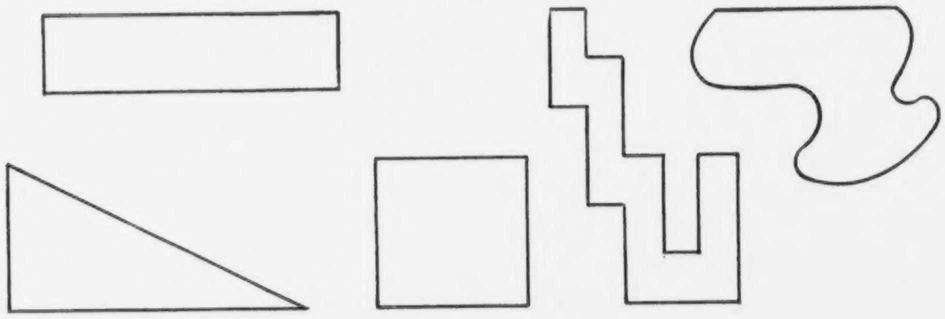
The child has been using, incidentally, inch-square shapes in activities involving comparison of areas. He may have used these as common units in order to compare areas effectively and to communicate his findings to others. His experience in measurement of length has made him aware of the usefulness of units of differing magnitudes. His



informal experience with area has led him to appreciate that both the magnitude and the shape of measuring units can facilitate measurement.

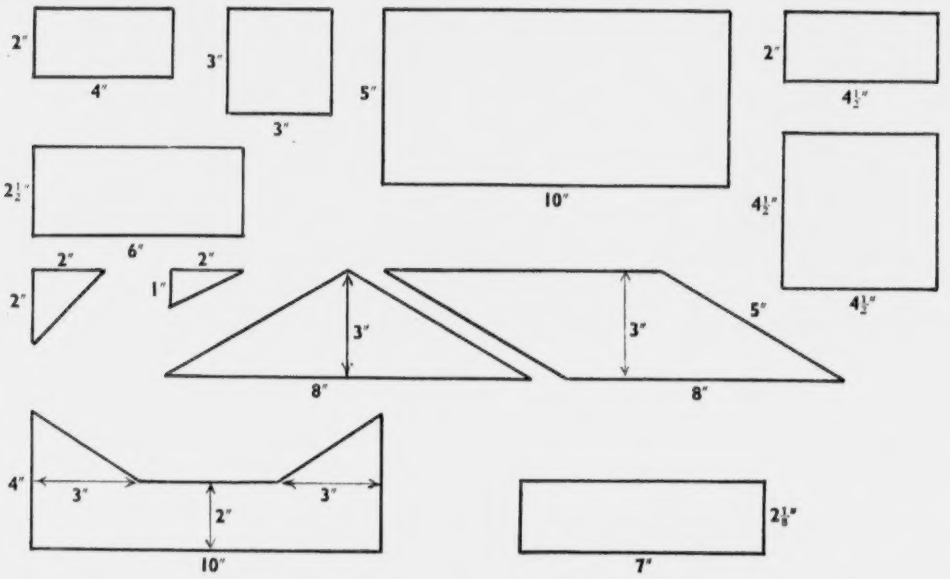
It is suggested that all three units—the square inch, the square foot, and the square yard—be introduced at the one time.

At the outset it is desirable to present a number of differently shaped regions, each of the same area—say, one square foot. The child is required to show that each of the shapes has the same area. He may be challenged to discover why the unit is called a square foot. Example: " Show that all of the shapes below are equal in area."



" This area is called I sq. ft. Why do you think this name was chosen ? "

The child should carry out many activities in which he measures regions by placing unit shapes over them. He may use inch squares or graticules (see Guide to Section G, Area, page 78) based on inch squares. Similar activities should be done using foot squares. Exercises should gradually increase in complexity. At first regions should contain an exact number of squares. Later, half squares could be utilised. Some regions should be of dimensions which will force the child either to estimate approximate measures or to cut the unit squares to measure more effectively. In all cases estimation should precede measurement. The following sequence of difficulty is suggested. Measures are included in the diagrams for teacher reference only. The child should be given an unmarked region and a supply of model units and/or graticules.



Numerous examples of each type suggested should be given and many other rectilinear shapes (figures bounded by straight lines) should be used. After a period of time, the child will not lay out all squares but will develop counting techniques, such as "number of rows × the number of squares in a row", to assist him to arrive at the measure of a rectangular region.

While activities involving square inches are being taken, a few exercises involving square feet and square yards may be given. Difficulty should match that experienced in examples involving square inches. Where areas do not measure exactly in the unit chosen the child should round off appropriately. After a few simple rectangular shapes are considered, more difficult rectangular shapes and other regions may be represented. For example :

Find in square feet :

(i) The area of a piece of flooring-board (4 in. wide) 12 ft. long.

(ii) The total surface area of a brick.

(iii) The area of the table-top.

When the child no longer needs to cover a rectangular or a triangular region with units he may discard them. It will be necessary, however, to continue to use a graticule in order to measure irregular regions.

The child should not confuse expressions such as "square inch" and "inch square". He must understand that the former does *not* describe shape, it is simply a name for a unit of measure. The latter, a descriptive term, describes a square region which has one-inch sides. The child should be aware, from his measurement activities, that a direct relationship does not necessarily exist between the measures of area and perimeter. The child should confidently explain the distinction between the statements "one square foot" and "one foot square" in terms of perimeters of each.

In this section, exercises involving complex computations of area should be avoided. The main objective is to gain deeper understanding of the measure of regions. The child will, as an outcome of experiences involving square inches, square feet, and square yards, come to realise that 144 sq. in. = 1 sq. ft.; and 9 sq. ft. = 1 sq. yd.

Exercises such as:

(i) 3¾ sq. ft. × 12 = □ sq. ft.
(ii) 3 sq. ft. 108 sq. in. × 12 = □ sq. ft.

have little value in promoting an understanding of area. By contrast the exercise: "How many tiles, ¾ in. square, are needed to cover a board 2 ft. long and 18 in. wide?" is quite appropriate.

In Section H, problems should allow, for the most part, a visualisation of the region concerned—perhaps in the form of a roughly scaled diagram. The child should still be able to have recourse to counting units. To this end the graticule, described in the Curriculum Guide, Section G, page 78, and the geoboard, based on an inch grid and using rubber bands to mark out regions, are useful aids.

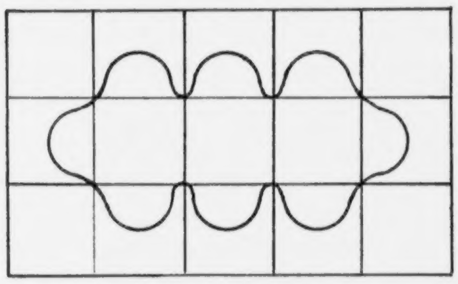
Some of the figures considered should be plane or "flat". Others should be parts of curved surfaces. Figures such as rectangles, triangles, cylinders, and triangular and square prisms should be included. The child will be familiar with the means of reducing the faces and the surfaces of prisms and cylinders to plane regions as a result of his experiences in spatial relations. In this form the area contained in the regions can be easily obtained.

SECTION I

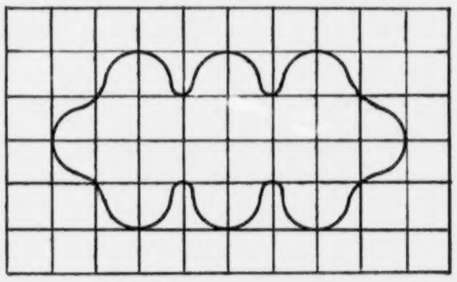
Understandings gained in earlier sections should be maintained. Estimation and subsequent measurement of rectangular and triangular regions should continue. Regions may become more complex in shape. Irregular and circular shapes will require the use of squares, either drawn or overlaid using some form of transparent material, in order to estimate area. Overlays can, of course, be used to measure area of rectangular or triangular shapes.

The child should be gaining an increasing understanding of the idea that greater precision of measurement is generally obtained by using smaller units. Measurements of continuous quantities such as area can only be an approximation to the actual size.

When the method of counting squares is applied, it is usual to count all squares more than half occupied by part of the region under consideration as whole squares. The squares not so filled are disregarded. Thus, for the region below, the following measures are obtained :



(i) 3 squares.



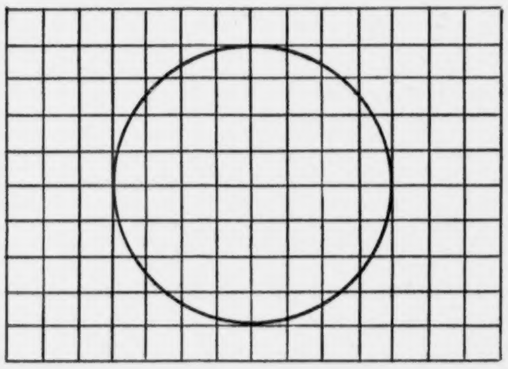
(ii) 28 squares.

The second measurement is equivalent to 7 large squares.

Smaller squares again might be used and a new approximation obtained. It would be useful for children to check successive approximations by taking paper replicas of the region and cutting and rearranging the pieces to fill completely as many squares as possible.

The illustrations shown above present the minimum reasonable over-estimate (28 small squares) and the maximum under-estimate (12 small squares) of the area of the region concerned. Some discussion of these limits could take place. Closer approximations lie between these two.

Using the superpositioning of squares, the areas of circular regions may be determined :



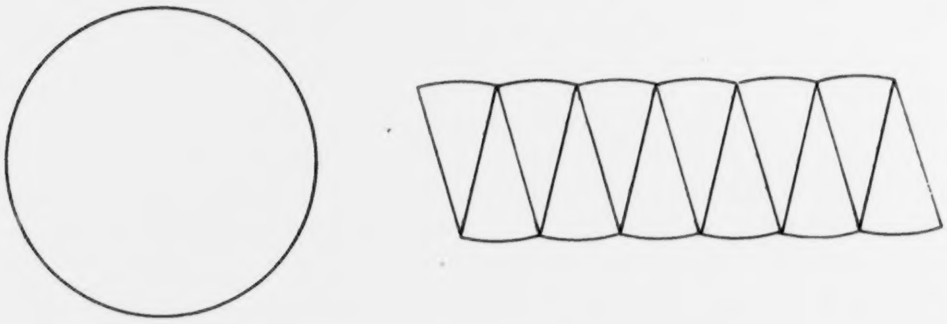
(*a*) One could adopt the above convention and arrive at an approximation of 52 squares.

(*b*) (i) If one were to choose only the complete squares included in the circle, above, the measure would be 32 squares.

(ii) If all squares, fully or partly enclosed by the circle, were counted the measure would be 60 squares.

Discussion will probably make plausible the suggestion that the best estimate will be half-way between these two limiting cases. In the above case the average or mean estimate would be 46 squares. (This activity can well be linked with the child's work in averages in this section.)

(c) A third effective method of obtaining a fair approximation to the area of a circle would be to divide the circle into a number of "equal" sectors. These are then cut out and rearranged to form a "parallelogram". A close approximation to the area of this new shape can readily be made.



VOLUME

SECTION H

The child should have had much experience with volume. He should by now have developed understandings of conservation of volume. These understandings should be developed by regular exercises involving estimation and subsequent checking. Where capacity is concerned he can check his estimates by using a formal measure such as the pint or the gallon. Early in Section H his estimation and measurement will be in terms of informal units such as bricks, wooden cubes, and cartons (for example, margarine boxes). The choice of a rectangular prism as a measuring unit has advantages that will have become apparent to the child as a result of his packing and stacking experiences. The further advantage of a cube-shaped unit should be brought out through directed activity and subsequent discussion.

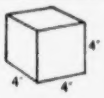
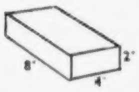
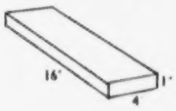
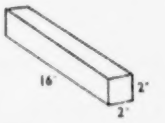
As in Length and Area the need for a standard formal unit of volume measurement is discovered. If a plentiful supply of inch cubes is available to the child, and if activities are directed to their use, little difficulty should be experienced in adopting the unit. The name "cubic inch" should follow naturally from square inch and inch.

Care must be exercised to ensure that the children distinguish clearly between cubic inch and inch cube. A cubic inch describes a volume and is independent of shape ; a cubic inch may be spherical, elongated, or any three-dimensional shape. An inch cube has a volume of one cubic inch and its shape is that of a cube with edges of one inch. An activity to assist this understanding would be to take clay or plasticine and ask the child to make an inch cube. Then suggest that he reshape the cube to form a ball or a long snake. Now have him reform the cube. Using the technique of displacement of water, discussed in Section G, the child can check that the volume of the cube and that of the reshaped object are in fact the same. Other rectangular prisms can be made from the cube.

Activities involving length of edge and area of rectangular solids can be undertaken. For example, using 64 inch cubes the table on page 87 can be built up.

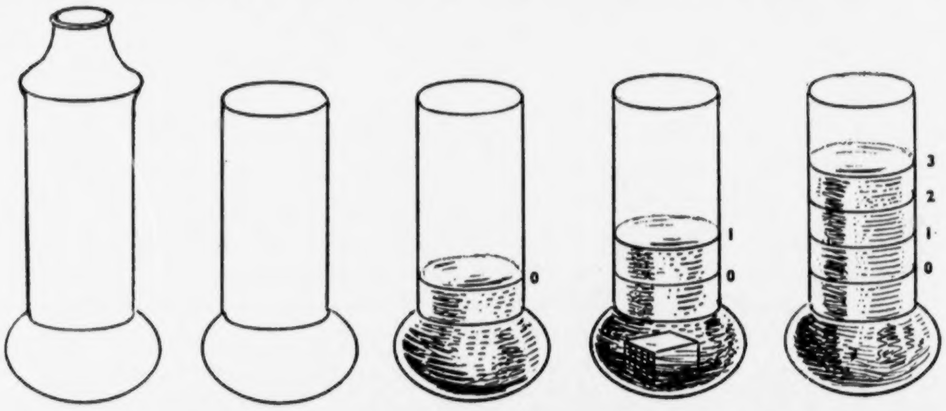
It is important that the child builds and counts for himself. It is also important that he should discuss with other children and/or the teacher what he proposes to do, is doing, or has done. Estimates of possible outcomes of activities should be discussed. For example, in the above activity the child might ask after the second rearrangement of blocks : "Is the measure of the sum of the areas of the faces always double the sum of the measures of the lengths of the edges ?" He might discuss the fourth result in the light of the first three, and proceed to a fifth situation in which he halves the height of the two-inch face and doubles the length of the shape.

Other activities involving inch cubes may be to construct a variety of shapes of differing volumes but having constant total edge length or constant total area.

Sketch of shape	Volume	Sum of areas of faces (by counting)	Sum of lengths of edges
(i) 	64 cu. in.	(16 + 16 + 16 + 16 + 16 + 16) = 96 sq. in.	48 inches
(ii) 	64 cu. in.	(16 + 32 + 16 + 32 + 8 + 8) = 112 sq. in.	56 inches
(iii) 	64 cu. in.	64 + 16 + 64 + 16 + 4 + 4 = 168 sq. in.	84 inches
(iv)  and so on	64 cu. in.	32 + 32 + 32 + 32 + 4 + 4 = 136 sq. in.	80 inches

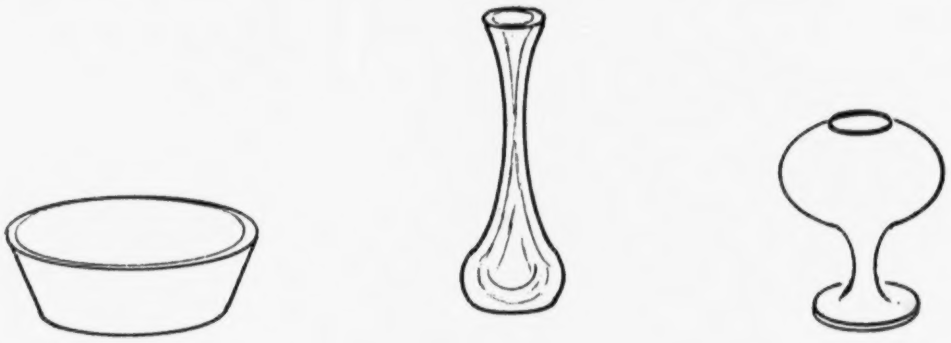
The child should be frequently challenged (and he should challenge others) to estimate volumes both in informal units and in cubic inches, and to use his ingenuity to check answers. Although estimation activities should generally involve shapes composed of rectangular prisms, some experiences involving other shapes are desirable.

Using tall, narrow vessels with parallel walls, simple measuring flasks may be constructed to read cubic inches. Take, for example, a plastic detergent container. Cut off the shaped top. Pour an inch or so of colored water into it and mark the level. Immerse a cubic-inch block and mark the new level. Fill to the new level with water and immerse the block again. Repeat the sequence two or three times. It will become evident that the graduations are equally spaced.



An activity involving internal volume would be to take a number of differently shaped vessels and attempt to estimate the volume of each in cubic inches. By means of a measuring jar such as that described above, the answers should be checked by measuring in cubic inches the volume of water that each vessel holds.

Again, take a vessel similar to one of those shown in the diagram below and attempt to estimate the successive levels to which water will rise each time a cubic inch of water is added.



SECTION I

Experiences should be provided in order to reinforce—

(i) the idea of conservation of volume ;

(ii) skill of estimation ; and

(iii) measurement of volume, using both informal and formal units.

In this section the cubic foot, the cubic yard, and the cubic centimetre are to be introduced.

It is suggested that a number of cube models with edges of one foot be made from cardboard. If each child were to make one such model, a class of 30+ children would have sufficient for initial experiences of measurement in cubic feet, and for making up one cubic yard.

It is probable that children who have reached Section I will be able to formulate a means for calculating the volume of rectangular prisms without the direct use of cubic units. They will have discovered that the shape is made up from a number of rows each containing a number of units to form a slice of unit height, and that a number of such slices form the total. Alternatively, the volume can be understood as a number of identical columns each of unit area in cross section and completely filling the space. Hence a foot cube contains twelve slices, each containing 144 units, or 144 columns each 12 units tall. In all, this is 1,728 inch cubes. Referring to his experiences of reshaping without change of volume, the child may generalise to say 1,728 cu. in. equals 1 cu. ft. Similarly 9 cu. ft. equals 1 cu. yd.

In the Cuisenaire material the white rod is a familiar shape with a volume of 1 cu. cm.

Four standard formal units of volume are now available to the child. In exercises he should now consider these and, unless directed otherwise, choose that which is most appropriate to the situation.

CAPACITY

SECTION H

The child, in earlier sections, has had much experience in estimation and measurement of the capacity of vessels. He is familiar with the units pint, quart, and gallon. (Little emphasis has been given to the quart.)

In Section H some attention should be given to estimation and measurement activities, and the child should show increasing accuracy in this work. Much attention should be given to formal processes and other computations involving pints, quarts, gallons, and the new unit to be introduced—the fluid ounce.

The Fluid Ounce

The child might be asked : " Does a fluid ounce have the same weight as an ounce dry weight ? " This question will lead to a variety of other questions and activities. For example :

(i) What is a fluid ounce ?
(ii) How many fluid ounces in one pint ?
(iii) How many ounces does a pint of water weigh ?
(iv) Is 1 fluid ounce a unit of capacity or of weight ?
(v) Does a pint of cooking oil have the same weight as a pint of water ?

Many of these questions can be answered by the child as a result of his direct measurement. Many containers should be labelled to show their volume in fluid ounces. The child might be encouraged to collect a variety of these—variety in terms of capacity and of shape.

Simple problems involving reduction of pints to fluid ounces and fluid ounces to pints can be undertaken.

Calculations Involving Units of Capacity

Equivalent measures involving fluid ounces, pints, quarts, and gallons can be written in fraction form. For example :

20 fluid ounces = 1 pint
1 pint = $\frac{1}{8}$ gallon
10 fluid ounces = $\frac{1}{2}$ pint or ·5 pint
60 fluid ounces = 3 pints or $\frac{3}{8}$ gallon.

The child might be presented with problems which can be solved by practical means or by recourse to the computational procedures with which he is equipped. On some (or many) occasions he can check formal calculation by practical measurement. For example :

(25 fl. oz. + $1\frac{1}{2}$ pints + 1·25 quarts) × 3 = $\square$
(1·25 pints + 1·5 pints + 2·5 pints) × 3 = $\square$
5·25 pints × 3 = 15·75 pints.

Alternatively, the three quantities could be measured out and the total found by multiplying the quantity obtained by three. Again, the child might take three of each measure independently and arrive at the same total amount.

Using measures of capacity, the child can quickly demonstrate the application of the basic properties of arithmetic. Consider the double application of the distributive property of multiplication over addition. For example :

$(14 + 6) \times (4 + 8) = \square$

Take $(14 + 6)$ fl. oz. $\times$ $(4 + 8) = \square$ fl. oz.

By the distributive property,

$\square = [14 \times 4 + 6 \times 4 + 14 \times 8 + 6 \times 8]$ fl. oz.

The child can take each of the quantities the stated number of times and readily find the total by tipping them together and measuring the result. By renaming $(14 + 6) \times (4 + 8)$ as 20×12, the child can obtain the result—$20 \times 12 = 240$; 20 fl. oz. $\times$ 12 = 240 fl. oz.—the same total as is obtained by the application of the distributive property.

Problems involving units of capacity should not—in terms of computation—present the child in Section H with much difficulty. Multipliers and divisors should be limited to whole numbers from 1 to 12, and reduction limited to two units.

Formal processes may well follow developments similar to those set out in Length, Section H.

Capacity provides a concrete aid to the development of fractions. The units are such that $\frac{1}{2}$s, $\frac{1}{4}$s $\frac{1}{8}$s, $\frac{1}{10}$s, $\frac{1}{5}$s, $\frac{1}{20}$s can be readily demonstrated.

4 pints $= \frac{1}{2}$ gallon
2 pints $= \frac{1}{4}$ gallon
1 pint $= \frac{1}{8}$ gallon

10 fl. oz. $= \frac{1}{2}$ pint $= \cdot 5$ pint
5 fl. oz. $= \frac{1}{4}$ pint $= \cdot 25$ pint
4 fl. oz. $= \frac{1}{5}$ pint $= \cdot 2$ pint
3 fl. oz. $= \frac{3}{20}$ pint $= \cdot 15$ pint
2 fl. oz. $= \frac{1}{10}$ pint $= \cdot 1$ pint
1 fl. oz. $= \frac{1}{20}$ pint $= \cdot 05$ pint

Thus a model is at hand to demonstrate operations with these fractions. For example :

(i) $1\frac{3}{8} + \frac{3}{4} = \square$
(ii) $\frac{4}{5} \times 5 = \square$
(iii) $1\frac{3}{4} \times 4 - \frac{3}{8} \times 3 = \square$
(iv) $1 \cdot 4 + \cdot 05 \times 6 = \square$
(v) $7 \cdot 35 \div 7 = \square$
(7·35 pints = 7 pints + 6 fl. oz. + 1 fl. oz.)

SECTION I

In his previous work the child has regularly practised estimation and measurement of the capacity of a variety of vessels. This form of experience should be continued. The child should realise that the capacities of vessels are measures of the volumes the vessels can contain. A fluid ounce, a pint, a millilitre, a bushel, . . . are units of volume, not of weight. A useful activity would be to find the weight of a bushel of each of various foods. Some of the weights could be found by inquiry. For example, bushels of apples or pears. Other weights could be found by weighing a pint or a gallon of the substance and then multiplying the result by the appropriate factor. (64 pints or 8 gallons are equivalent to 1 bushel.) An investigation into the method employed to find the weight of a bushel of wheat (f.a.q.) for any one year in Victoria can be made with profit. Investigations and activities of this kind can supply useful data for graphical representation.

The multipliers and the divisors used in process work may be extended to whole numbers up to and including 100. Where addition, subtraction, multiplication, or division is involved, the child should use the method of solution he finds most appropriate to the particular problem. This may be doubling and/or halving, renaming in unconventional ways, or restating the problem in some way. The formal process is frequently the best way of meeting the situation. The development of formal processes is set out in the Guide to Section H, Length. Those for capacity may well be developed along similar lines.

WEIGHT

SECTION H

The child is familiar with the units ounce, pound, and stone. He should be able to associate the weighing of various objects with particular units. He might associate chocolate with ounces, bricks with pounds, and children with stones. He should be encouraged to consider various objects in terms of appropriate units. The ton has been introduced and the child possibly associates this with firewood or with motor-cars. Some children will be familiar with the ton associated with metal sales or with fertilisers. The child might be asked to write the names of appropriate units for the measurement of the respective weights of an elephant, a mouse, a cat, a goat, a motor mower, a motor-truck, or a wrist watch.

Experience has enabled the child to realise that size is not necessarily a reliable cue for estimating weight. His experience should be broadened to involve a wide range of materials, and he should be developing more accuracy in his estimation of weight. Given various objects that have equivalent sizes and shapes, the child might be asked to order them according to weight simply by lifting or holding them. The child can check his results by direct weighing. Suitable objects would be a brick, a red-gum block, a pine block, a hardwood block, a foam-rubber block, and a polystyrene block. Some attention might be given to the relative weights of metals, say, lead, iron, brass, and aluminium. Estimation activities may also involve the use of identical cartons filled with materials of different densities.

In his direct weighing activities the child might be asked to find the weight of one of a large number of small identical objects such as paper clips. To do this he might find the weight of a large number, say 100, and calculate the weight of one clip by division.

The quarter and the hundredweight are introduced in Section H. The child should appreciate the need for units between the stone and the ton.

An investigation into the historical development of each (quarter, hundredweight) may show that, at one time, many goods were measured in quarters or hundredweight. The quarter was a popular unit for provisions such as potatoes. The hundredweight was a useful quantity of firewood or coal when people could not, for various reasons (economic, storage, cartage), buy ton lots. The hundredweight is still a popular unit for fuel measurement and in the measurement of the weight of trucks and other vehicles.

The table of weights as set out in the Course of Study should be known. Problems involving reduction exercises should not be unduly complex and should not, in general, involve more than two units. For example :

(i) How many hundredweight lots of wood can be obtained from 13 tons 5 cwt ?

(13 × 20 + 5) cwt.
(260 + 5) cwt.
Answer : 265 lots.

(ii) How many pound packets of sugar can be obtained from a consignment of $2\frac{1}{2}$ cwt ?

$2\frac{1}{2}$ cwt. = (2 × 112 + 56) lb.
= (224 + 56) lb.
= 280 lb.
Answer : 280 packets.

(iii) Rename 15 cwt. 65 lb. in pounds

$$\begin{array}{llll}
 & 15 \text{ cwt. } 65 \text{ lb.} & \textit{or} & 15 \text{ cwt. } 65 \text{ lb.} \\
 & 15 \times 112 + 65 & & \times\ 112 \\
= & 30 \times 56 + 65 & & \overline{30} \\
= & 1680 + 65 & & 150 \\
= & 1680 & & 1500 \\
 & +\ 65 & & \overline{1680 \text{ lb.}} \\
 & \overline{1745} & & +\ 65 \text{ lb.} \\
 & & & \overline{1745 \text{ lb.}}
\end{array}$$

Answer : 1745 lb.

(iv) How many bags of onions, each holding one quarter, can be filled from a truck containing 2,000 lb. ?

2000 ÷ 28 = ☐ qr. ☐ lb.

$$\begin{array}{r|r|r}
28 & 2000 \text{ lb.} & \\
 & 1400 & 50 \text{ qr.} \\
\hline
 & 600 & \\
 & 560 & 20 \text{ qr.} \\
\hline
 & 40 & \\
 & 28 & 1 \text{ qr.} \\
\hline
 & 12 \text{ lb.} & 71 \text{ qr.}
\end{array}$$

Answer : 71 bags, 12 lb. remainder.

Frequently a computation is simplified if the weights involved are expressed in fractional form. In some cases weight can be used to assist children to understand fractions and operations involving fractions. In this latter case weights used should be of suitable size to enable direct checks by weighing in the classroom. (See Curriculum Guide, Section H, Equations.)

Tables such as the following might be built up :

(*a*) 1 ton = 20 cwt. = ☐ quarters = 2,240 lb.
½ ton = ☐ cwt. = 40 qr. = ☐ lb.
¼ ton = ☐ cwt. = ☐ qr. = ☐ lb.
☐ ton = 2½ cwt. = ☐ qr. = 280 lb.

(*b*) 1 cwt. = 4 qr. = 112 lb.
½ cwt. = 2 qr. = ☐ lb.
☐ cwt. = 1 qr. = ☐ lb.
☐ cwt. = ½ qr. = 14 lb.

A number of approaches to the solution of equations was suggested in the Guide to Section H, Length. In this topic (Weight) similar comments apply. The child should have a variety of means whereby he can carry out required computations, and he should choose the most appropriate to suit the problem in hand. Various suggestions as to methods of computation are made in the topic Length. For example, the renaming of units in decimal or vulgar fractions, the restatement of the problem in more convenient terms, doubling and/or halving techniques, and the formal process. Reference to Length will show these techniques set out in worked examples.

SECTION I

The bushel or its approximate equivalent has been used as a measure of grain since ancient times. One theory for the early appearance and the persistence of the bushel is that this amount of wheat is about the load a man can carry over appreciable distances.

The Imperial Standard Bushel was defined by the Weights and Measures Act of 1824 as the equivalent of 8 gallons or the volume of 80 pounds of water.

As the bushel is a measure of capacity, and since it was used generally for dry provisions such as wheat, apples, green beans, tomatoes, and onions, an interesting discussion can develop as to the value or the effectiveness of the bushel as a unit of measure. The child might investigate current practices in the measurement of provisions such as those listed above.

Clearly, one cannot justify the equating of a bushel to a specific weight. Some gross approximations of the weight of a bushel of various commodities are :

1 bushel of wheat is about 60 pounds weight.
1 bushel of apples is about 45 pounds weight.
1 bushel of onions is about 55 pounds weight.
1 bushel of barley is about 50 pounds weight.
1 bushel of oats is about 40 pounds weight.

It is interesting to note that a bushel of wheat may vary from as low as 53 lb. up to 70 lb., depending on the quality of the sample. Each year, in Victoria, a standard weight of fair average quality (f.a.q.) wheat is nominated. The procedure used for obtaining this standard weight is interesting, and inquiry into it would prove a useful project for children in Section I. (In practice, the produce merchant sells wheat in bushels of 60 pounds.)

The short ton is used as standard practice in Australia as the unit for measuring all mill products (flour, bran, pollard). This ton is equal to 2,000 lb. In North America the short ton is the standard ton ; in Australia, the United Kingdom, and other Commonwealth countries (except Canada) the long ton of 2,240 lb. is the standard measure for substances other than mill products. (In North America the hundredweight is 100 pounds).

The child's development of estimation of weight should be maintained, and efforts should be made to further develop this skill.

Reduction exercises should not, in general, involve more than two units. Such exercises will be more meaningful and interesting to the child if they are outcomes of realistic problems. For example :

(i) Is it more economical to purchase 4 four-ounce packets of tea at 16c each or to buy a pound packet for 60c ?

(ii) If a baker uses $1\frac{1}{2}$ lb. of flour to make a loaf of bread, how many loaves can he make from $\frac{3}{4}$ ton of flour ?

Process work involving weight should generally arise from problems. Although a formal process should be developed and understood, the child in Section I should be encouraged to investigate alternative means of solution for each problem. For example : How many tons of wheat are transported by a train of 28 trucks if each truck contains 21 tons 15 cwt ? Various ways of attacking this problem are set out below :

(i) 21 tons 15 cwt. × 28

21 tons	15 cwt.
× 28	× 28
168	120
420	300
588 tons	420 cwt.

= 588 tons + 21 tons

Answer : 609 tons.

(ii) 21 tons 15 cwt. × 28
= (420 + 15) cwt. × 28
= 435 cwt. × 28

435
× 28
3480
8700
12,180

20	12180	
	12000	600
	180	
	180	9
		609

Answer : 609 tons.

(iii) 21 tons 15 cwt. × 28
= (22 tons − 5 cwt.) × 28
= 22 tons × 28 − 5 cwt. × 28
= 616 tons − 140 cwt.
= 616 tons − 7 tons
= 609 tons

28
× 22
56
560
616

28
× 5
140

Answer : 609 tons.

(iv) 21 tons 15 cwt. × 28
= (21 tons 15 cwt.) × (30 − 2)
= 630 tons + 22 tons 10 cwt.
− (42 tons + 1 ton 10 cwt.)
= 652 tons 10 cwt.
− 43 tons 10 cwt.

Answer : 609 tons 0 cwt.

21 × 30 = 630
15 × 30 = 450
450 ÷ 20 = 22, rem 10

(v) 21 tons 15 cwt. × 28
$21\frac{3}{4} \times 28$
$= \frac{87}{4} \times 28$
$= 87 \times \frac{28}{4} = 87 \times 7$
= 609
Answer : 609 tons.

(vi) 21 tons 15 cwt. × 28
21 tons 15 cwt.
43 tons 10 cwt. (2 times)
87 tons 0 cwt. (4 times)
174 tons 0 cwt. (8 times)
348 tons 0 cwt. (16 times)

696 tons 0 cwt. (32 times)
− 87 tons 0 cwt. − (4 times)

609 tons 0 cwt. (28 times)

Answer : 609 tons.

TIME

SECTION H

The child has discovered that units of length, area, volume, capacity, and weight have been quite arbitrarily chosen. Furthermore, there is no world-wide adoption of any of them. On the other hand the major units of time are virtually accepted by all concerned with the measurement of time. These units are based on the day (the period of a rotation of the earth), the month (the period of one revolution of the moon around the earth), and the year. A difficulty exists however in the fact that there is not an exact number of days in the lunar month nor in the period of revolution of the earth about the sun. Furthermore, there is not an exact number of lunar months in this period. The calendar we use has been developed to account for the 365¼ days (more precisely, a little less than a quarter day—5 hr. 48 min. 46·43 sec.).

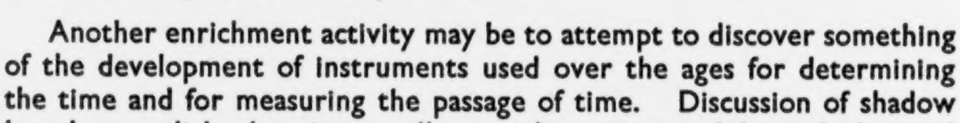
In Section H the child should be aware that the somewhat arbitrary break-up of the year into calendar months of various durations, and into weeks, is a convenient way of recording the passage of time. In all other measurement experiences he has found sub-units to be aliquot parts of a major unit.

As an enrichment activity some children may investigate the history of the development of today's calendar.

Another enrichment activity may be to attempt to discover something of the development of instruments used over the ages for determining the time and for measuring the passage of time. Discussion of shadow lengths, sundials, burning candles, sand-timers, pendulum clocks, and spring clocks may result. The use of weights as driving forces for some types of clocks might be considered.

The child should be familiar with the terms "a.m." and "p.m.". The day is divided into 24 hours—the period from midnight to midnight. The mid-point of this period is that time when the sun is at its meridian. At this time (for true midday) the sun is at its highest point in the sky and is due north of the observer. Times before midday are referred to as *ante meridiem* (a.m.) and times after midday as *post meridiem* (p.m.). The child should be given opportunity to state the time in terms of a.m. or p.m. at various times throughout the day until he is quite familiar with the notation.

The term "century" should be introduced. The difficulty frequently experienced in relating the particular period to its position in the overall time scale may be overcome by considering the child's age and relating it to an ordinal scale. Thus an eleven-year-old child is in his twelfth year. The number line (time line) can also assist in this way.



The twentieth interval represents the twentieth century. From the number line it is seen that the twentieth interval is that from 1900 to 2000.

Reduction

Reduction exercises involving time should not, in general, involve more than two units. As a result, reduction exercises will simply be a matter of multiplication or division by the appropriate number.

The reduction exercise will generally be incidental to the solution of problems. Simple oral exercises should be frequently taken to ensure that the relevant tables are known and that the child is confident in regard to the operation he should use. Problems involving reduction might be as follows:

> A train travelled 1·2 miles in one minute. How many miles would it travel in two hours ten minutes at the same speed ?

The child might say:

1·2 miles in 1 minute
2·4 miles in 2 minutes
24 miles in 20 minutes
72 miles in 60 minutes (1 hour)
144 miles in 2 hours
+ 12 miles in 10 minutes
156 miles in 2 hours 10 minutes
Answer: 156 miles.

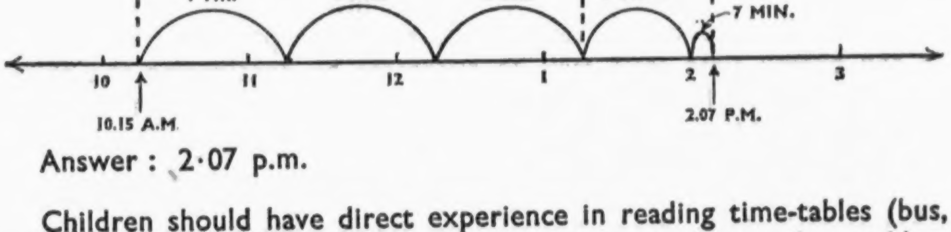
or

2 hr. 10 min.
= 120 min. + 10 min.
= 130 minutes
130 × 1·2
= 13 × 12 = 144 + 12
= 156
Answer: 156 miles.

Addition and Subtraction

Adding or subtracting measures of time may be carried out in the ways suggested for adding and subtracting measures of length (see pages 70–73 of this Guide).

When time intervals are added to or subtracted from a specific time a number line can be used to demonstrate the situation. For example: A bus arrived at OZ at 10.15 a.m. and departed 3 hours 52 minutes later. At what time did it depart ?



Answer: 2·07 p.m.

Children should have direct experience in reading time-tables (bus, train, air, ship) and should carry out activities similar to the problem above from time-table data.

Multiplication and Division

As has been suggested in other topics, multiplication and division processes involved in the solution of problems should be examined and subsequently solved by the form most appropriate to the situation. Frequently this will be by a formal procedure.

Consider the problem : A train driver made 9 trips each of 3 hours 35 minutes. What was his total driving time ?

(i) 3 hr. 35 min.
$\times$ 9
27 hr. 315 min.

= 27 hr. + 300 min. + 15 min.
= (27 + 5) hr. 15 min.
= 32 hr. 15 min.
Answer : 32 hr. 15 min.

(ii) 3 hr. 35 min.
$\times$ 9
27 hr. 315 min.

27 hr. 60 | 315 min. | 5 hr.
300 min.
15 min.

27 + 5 hr. 15 min.
32 hr. 15 min.
Answer : 32 hr. 15 min.

(iii) 3 hr. 35 min. $\times$ 9
$= (3\frac{35}{60} \times 9)$ hr.
$= (3\frac{7}{12} \times 9)$ hr.

$$= \frac{43}{12} \times \frac{9}{1} = \frac{43 \times 3 \times 3}{4 \times 3} = \frac{43 \times 3}{4} \times \frac{3}{3} = \frac{129}{4} = 32\tfrac{1}{4}.$$

Answer : 32 hr. 15 min. ; or $32\frac{1}{4}$ hr.

Consider a problem involving division (partition) : Over a period of 20 weeks and 4 days, eight men equally shared fire-watch duties from a tower in the forest. How much time did each man spend on watch ?

20 weeks 4 days $\div$ 8.
(i) (16 + 4) weeks 4 days $\div$ 8
= (16 weeks + 4 weeks 4 days) $\div$ 8
= (16 weeks + 32 days) $\div$ 8
Answer : 2 weeks 4 days

(ii) 8 | 20 weeks 4 days | 2 weeks
16 weeks
4 weeks
$\times$7
28 days
+4 days
32 days
32 days
0 | 4 days

Answer : 2 weeks 4 days

(iii) 20 weeks 4 days $\div$ 8
$(20\frac{4}{7} \div 8)$ weeks
$\frac{144}{7} \div 8$ weeks
$\frac{144}{7} \times \frac{1}{8} = \frac{18}{7}$ weeks
$= 2\frac{4}{7}$ weeks
Answer : 2 weeks 4 days ;
or $2\frac{4}{7}$ weeks

Consider a problem involving quotition division : The time interval between the beginning of one ride and the beginning of the next on a circus merry-go-round was $2\frac{1}{2}$ minutes. How many rides could have been taken over three hours ?

3 hours $\div 2\frac{1}{2}$ minutes.

180 minutes $\div 2\frac{1}{2}$ minutes

(i) $180 \div 2\frac{1}{2}$

$360 \div 5$

72

Answer : 72 rides

(ii) $180 \div 2\frac{1}{2}$

$= 180 \div \frac{5}{2}$

$= 180 \times \frac{2}{5}$

$$= \frac{180 \times 2}{5}$$

$$= \frac{36 \times 5 \times 2}{5}$$

$= 72$ rides

Answer : 72 rides

Telling the Time, Recording the Date, Estimation of Time Intervals

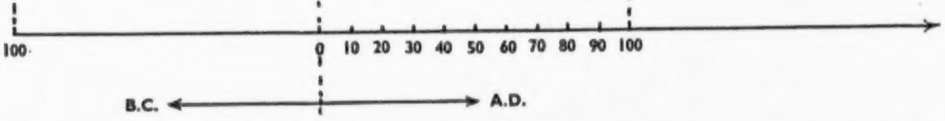
Skills developed in previous sections should be maintained through regular activities. The child should strive for greater precision when the activity concerned warrants this, for example, in timing team games or foot-races.

SECTION I

Understandings and skills developed in Section H should be maintained and further developed or refined in Section I.

The Calendar

The child's knowledge should be extended to include the notion of the reference point—the birth of Christ—and the notation B.C. and A.D. Again the number line is a useful model :



The child will realise, by reference to two or three calendars of successive years, that Easter does not fall on the same date each year. Reference to a dictionary will enable him to discover that Easter Sunday is that Sunday which first occurs after the first full moon following March 21 in any year. This formula may then be checked by reference to calendars of various years.

The child is probably aware, from his attention to radio broadcasts of events taking place in other countries (for example, Test cricket, Davis Cup tennis, boxing contests), that at any one moment the stated time varies from place to place.

Frequent recourse to a globe may assist him to understand the relationship between rotation, longitude, and the time of day. The need for a convention will become obvious ; without a convention all adjacent places east or west of one another would work to different times. A reference line is required and broad divisions need to be adopted. The child, using the knowledge that the earth rotates once in twenty-four hours, may be able to discover that each 15 degrees of longitude corresponds to $\frac{1}{24}$ of one rotation, or one hour.

If corresponding times in Australia are then considered, it will be found that Western Australian time is two hours behind Victorian time, while in South Australia the lag is only half an hour. It will be discovered, too, that Victoria, New South Wales, and Queensland use the same time (Eastern Standard Time). Some light can be thrown on the problem by studying a world map showing standard-time zones. The need for reference lines such as the meridian through Greenwich and the International Date Line will become clearer as a result of discussion following study of the map.

In this topic, as in all topics in Section I, the depth of treatment for individual children will depend on the ability and the interests of the child. For example, only a partial understanding of the time zones is likely to be achieved by the majority of children at this level.

MONEY

SECTION H

Recognition of All Coins and Notes

In most cases children at this level are quite familiar with all the coins and the notes in use. Teachers should assure themselves that such is the case. In some localities where money is not frequently handled in direct transactions, children may need opportunities to observe and handle money in the classroom. Some discussion with all children may be carried out so that they become familiar with the motifs and other details of significance on notes and coins. Activities may also involve the weighing of coins in an endeavor to find such relations where these exist between coins of different denominations. The child might weigh 10 ten-cent coins, 5 twenty-cent coins, 5 two-cent coins, and 10 one-cent coins. An investigation into the methods used by banks in counting money and packaging it would prove worth while and interesting.

Writing Amounts in Dollars and Cents

Maintenance and strengthening of work done in Section G should be carried out. (See Curriculum Guide, Section G, pages 97–102.)

Money Relationships

The development as set out in Curriculum Guide, Section G, pages 98, 99, should be revised and extended to involve decimal fractions. For example :

$$\left.\begin{array}{l}1\text{c}\\ \$0{\cdot}01\end{array}\right\} = \tfrac{1}{100}\text{ dollar} = {\cdot}01\text{ dollar}$$

$$\left.\begin{array}{l}7\text{c}\\ \$0{\cdot}07\end{array}\right\} = \tfrac{7}{100}\text{ dollar} = {\cdot}07\text{ dollar}$$

$$\left.\begin{array}{l}10\text{c}\\ \$0{\cdot}10\end{array}\right\} = \tfrac{10}{100}\text{ dollar} = {\cdot}10\text{ dollar}$$

$$\left.\begin{array}{l}14\text{c}\\ \$0{\cdot}14\end{array}\right\} = \tfrac{14}{100}\text{ dollar} = {\cdot}14\text{ dollar}$$

$$\left.\begin{array}{l}89\text{c}\\ \$0{\cdot}89\end{array}\right\} = \tfrac{89}{100}\text{ dollar} = {\cdot}89\text{ dollar}$$

$$\left.\begin{array}{l}231\text{c}\\ \$2{\cdot}31\end{array}\right\} = \tfrac{231}{100}\text{ dollar} = 2{\cdot}31\text{ dollars}$$

Processes

At some time during this section the processes involving money should become closely allied with those in decimal fractions. This approach involves the understanding by the child that any amount in decimal currency can be expressed as a decimal fraction of the dollar. For example, 13 dollars 78 cents × 79 may be rewritten as $13·78 × 79, and the operation carried out as for decimal fractions.

It is emphasised, as is the case in other topic areas, that the process should not form the stock approach to the solution of equations. The child should handle formal processes with confidence, but having achieved facility in them he should not discard other techniques that have been developed for the solution of equations.

The Course of Study provides for :

(i) Oral response to simple operations involving amounts to $5;

(ii) recorded response to operations involving total amounts to $100 ;

(iii) multipliers to 100 and divisors to 20 ; and

(iv) activities leading to short division of money with divisors to 10.

Division may or may not involve remainders. Consider the following examples :

(i) $25·12 ÷ 16 = ☐

Rename $25·12 as 2512c

$$\begin{array}{r|r|r} 16 & 2512 & \\ & 1600 & 100 \\ \hline & 912 & \\ & 800 & 50 \\ \hline & 112 & \\ & 112 & 7 \\ \hline & 0 & 157 \end{array}$$

Answer: $25·12 ÷ 16 = $1·57

(ii) $25·12 ÷ 15

Rename $25·12 as 2512c

$$\begin{array}{r|r|r} 15 & 2512 & \\ & 1500 & 100 \\ \hline & 1012 & \\ & 900 & 60 \\ \hline & 112 & \\ & 105 & 7 \\ \hline & 7 & 167 \end{array}$$

Then $1·67 × 15 + 7c = $25·12
Answer: 167c, 7c remainder; or $1·67, remainder 7c

Problems such as : " How many magazines each costing 18c can be bought with $1·26 ? " can be attacked (i) by successive addition or subtraction, or (ii) by renaming the amounts concerned in like units and carrying out division with whole numbers. Thus :

(i)
18c (1 magazine)
36c (2 magazines)
72c (4 magazines)
108c (6 magazines)
$1·26 = 126c (7 magazines)
Answer : 7 magazines

(ii)
$1·26
−0·18c (1)
$1·08
−0·18c (1)
90c
− 18c (1)
72c
− 18c (1)
54c
− 18c (1)
36c
− 18c (1)
18c
− 18c (1)
0
Answer: 7 magazines

(iii) Rename $1·26
as 126c
126 ÷ 18 = ☐

$$\begin{array}{r|r|l} 18 & 126 & \\ & 108 & 6 \\ \hline & 18 & \\ & 18 & 1 \\ \hline & 0 & 7 \end{array}$$

Answer: 7 magazines

Short division relies on the property of distribution of division over addition (and subtraction). The child has used this property with whole numbers. In his short division activities the child should rename amounts of money in ways that facilitate division. For example :

(i) $21·07 ÷ 7
= ($21 + 7c) ÷ 7
= $21 ÷ 7 + 7c ÷ 7
= $3 + 1c
= $3·01

$21·07 ÷ 7 = $3·01

(ii) $22·05 ÷ 7
= ($21 + $1·05) ÷ 7
= ($21 + 105c) ÷ 7
= ($21 + 70c + 35c) ÷ 7
= $3 + 10c + 5c
= $3·15

$22·05 ÷ 7 = $3·15

(iii) $27·93 ÷ 7
= ($28 − 7c) ÷ 7
$4 − 1c
= $3·99

$27·93 ÷ 7 = $3·99

(iv) $33·04 ÷ 7
= ($28 + $5 + 4c) ÷ 7
= ($28 + $3·50 + $1·50 + 4c) ÷ 7
= ($28 + $3·50 + 140c + 14c) ÷ 7
= $4 + 50c + 20c + 2c
= $4·72

$33·04 ÷ 7 = $4·72

The following list of exercises shows a suitably graded sequence. Many examples at each level would be needed. The child may be able to do some exercises without recording his procedural steps.

(i) $18·00 ÷ 6
(ii) $18·06 ÷ 6
(iii) $18·66 ÷ 6
(iv) $19·08 ÷ 6
(v) $33·00 ÷ 6
(vi) $34·14 ÷ 6

Problems

Shopping activities, within the limit suggested, should include both oral and recorded work and should continue to provide the basis for process work *in decimal currency.* These activities may include listing up to four items, totalling the prices, and working out the required change. Transactions may involve payment and the receipt of change for train, tram, or bus tickets (old tickets are useful), telephone calls, telegrams, stamps, admission tickets (football), and so on. The school bank, social service, and excursion and other levies provide useful environmental situations that can be exploited.

In many cases children at this level are interested in how much can be obtained from spending an amount of money rather than in the amount of change they will obtain as the result of a transaction. Consequently, examples of the following kind are useful :

> Frances has $1·50 to spend at the pet shop. The animals that interest her are white mice (30c each) ; guinea-pigs (45c each) ; blue-tongued lizards (80c each) ; gold-fish (24c each) ; water-snails (2c each) ; white rabbits (95c each) ; and tortoises (55c each).
>
> (i) What do you think you would buy if you were Frances ?
>
> (ii) Record your purchases and show how much you would spend altogether.

The environmental problem expressed in words is clarified if it can be translated into symbolic terms, that is, an incomplete equation. It may be useful in many cases to develop a midway step in which a combination of everyday language and mathematical symbols is used. Consider the following example :

> John has $1·00 per week allowance. From this amount he pays bus fares to and from school (7c each way), a club fee of 20c each week, and he gives 5c to social service each Friday. He walks home from school on three occasions each week.
>
> (i) If John has no other expenses, how much can he save each week ?
>
> (ii) How long would it take to save $15·86 to buy a new watch ?

This problem could be a group effort. Arising from discussion, the following ideas might emerge :

(i) John uses the bus seven times a week ; he pays 20c for a club fee ; he gives 5c to social service.

Thus John spends 7 times 7c, 20c, and 5c

$= 7c \times 7 + 20c + 5c$

$= 49c + 20c + 5c$

$= 74c$

John has ($1·00 — 74c) left. $74 + (6 + 20) = 100$.

He can save 26c each week.

(ii) How many 26c are needed to make \$15·86 ?

One child might say : $26c \times \square = \$16{\cdot}00$

and $26c \times 2 = 52c$

$26c \times 4 = 104c = \$1{\cdot}04$

$26c \times 8 = 208c = \$2{\cdot}08$

$26c \times 16 = 416c = \$4{\cdot}16$

$26c \times 32 = 832c = \$8{\cdot}32$

$26c \times 64 = 1664c = \$16{\cdot}64$

$26c \times 63 = 1638c = \$16{\cdot}38$

$26c \times 62 = 1612c = \$16{\cdot}12$

$26c \times 61 = 1586c = \$15{\cdot}86$

so it would take 61 weeks to save the \$15·86.

Another child might say : "How many times can I take 26c from \$15·86 ?

In equation form $1586c \div 26c = \square$

$$\begin{array}{r|r|l} 26 & 1586 & \\ & 1300 & 50 \\ \cline{2-2} & 286 & \\ & 260 & 10 \\ \cline{2-2} & 26 & \\ & 26 & 1 \\ \cline{2-3} & 0 & 61 \end{array}$$

Hence, $1586 \div 26 = 61$

or $26c \times 61 = 1586c$ or \$15·86

Answer : 61 weeks

Teachers should ensure that the vocabulary used in problems is readily understood by the child. The reading level and the complexity of the problem must both be related to the capacity of the individual. Some of the difficulty in relating the problem to the child may be overcome by allowing some examples to become group problems (as was the case in the example worked above).

The child should develop the practice of checking the feasibility of his answers. This could be assisted by considering the problem in group discussion prior to the actual working by individuals. As an outcome of discussion, an estimated answer may be obtained. The child might be required at other times to estimate an answer prior to his step-by-step computation, and to write this answer down so that a comparison can be made later between his estimated answer and the calculated answer.

Children should be encouraged to compare prices. For example :

(i) Lemonade is to be bought for a party. Is it more economical to buy the super-size brand containing 32 fluid ounces at 24c a bottle than it is to buy another brand containing 30 fluid ounces at 20c a bottle ?

(ii) Louise keeps budgerigars. A packet of seed costs 28c. The packet lasts two weeks. Another brand costs 36c, but this lasts three weeks. If she were to buy the cheaper brand (that is the one which gives most seed for the same money) how much would Louise have to set aside from her pocket-money each week to buy seed for her birds ?

(iii) Linda went swimming twice a week for fourteen weeks. Admission to the pool was 20c on each occasion. She walked to the pool on each occasion and took a tram home. The tram fare was 7c. How much did Linda spend altogether on swimming and travelling home from the pool ?

(iv) Bruce buys his lunch each day from the school canteen. If, for three weeks, he bought a meat pie (13c), an apple (4c), a buttered bun (6c), and a half-pint bottle of milk (6c) each day, how much did he spend on lunches during this time ?

(v) Alan wants a new bicycle. He could buy one for $38 cash or he could pay $10 deposit and $2·75 per month for twelve months. How much more would the bicycle cost if it was bought on the time-payment plan than it would if he saved up and paid cash for it ?

SECTION I

Understandings and skills developed in previous sections should be maintained and strengthened.

Amounts of money involved in computations may be increased as set out in the Course of Study (oral exercises to $10 ; recorded exercises to $1000 ; and multipliers and divisors to 100).

Where amounts of money are to be partitioned, and a fraction of a cent results in the answer, some confusion frequently develops. Discussion on this outcome should be undertaken and should be related to an actual problem. For example : $27·30 is to be shared equally among 4 boys. How much should each boy receive ?

$$\$27{\cdot}30 \div 4 = \square$$

This is completed as $\$27{\cdot}30 \div 4 = 6{\cdot}825$ dollars (or 6 dollars 82½ cents). In this case we can write the answer as $6·82, with $(\frac{1}{2} \times 4) = 2$c rem. ; or as $6·82 to each boy (2 cents remainder).

Where an amount of money is to be divided by another amount of money, the answer is a number and not an amount of money—the quotient may be a whole number, a fraction less than one, or a combination of these. The answer will depend on the particular problem. Frequently a fractional part less than one is unacceptable.

For example :

(i) How many eggs at 7c each can be bought for 50c ? (Answer : 7 eggs.)

(ii) How many weeks will it take a boy to save $2 if he saves 24c each week ? (In 8 weeks he would save $1·92. Hence 9 weeks would be needed to save $2.)

On some occasions an answer with a fractional part is meaningful, for example : How many yards of material at 88c a yard can be bought for $4·84 ? The answer is $5\frac{1}{2}$ yards.

Some problems involving division of money involve remainders where part of a cent is not a meaningful part of the solution and, consequently, a number of cents less than the divisor remains. How can such a problem be expressed in equation form ? Consider the example : $72·24 ÷ 25

This can be written as $\square \times 25 + \triangle = \$72{\cdot}24$.

This means that the quotient ($\square$) multiplied by 25, plus the remainder ($\triangle$) equals $72·24.

$$\begin{array}{r|r|l} 25 & 7224 & \\ & 5000 & 200 \\ \cline{2-2} & 2224 & \\ & 2000 & 80 \\ \cline{2-2} & 224 & \\ & 200 & 8 \\ \cline{2-3} & 24 & 288 \end{array}$$

We can write : $\$2{\cdot}88 \times 25 + 24c = \$72{\cdot}24$.

The answer to the problem may be written as $2·88, rem. 24c.

Problems may involve the preparation of simple invoices. Attention should be directed to the actual transactions rather than to the printed details of the invoice. It would be useful if actual invoice forms were obtained and some work done directly with these.

Percentage was introduced in Section H. In that section the child was asked to express simple vulgar and decimal fractions as percentages. Now he can express smaller amounts of money as percentages of larger amounts. Care should be taken to ensure that the percentage does not involve fractional parts or mixed numbers (e.g. $7\frac{1}{2}\%$). Suitable exercises would be as follows :

(i) Express 7c as a percentage of $1·00
$7c = \frac{7}{100}$ of $\$1 = 7\%$ of $1

(ii) Express 9c as a percentage of $3·00
$9c = \frac{9}{300}$ of $3·00
$= \frac{3}{100}$ of $3·00
Then $9c = 3\%$ of $3·00

(iii) $15\% \text{ of } \$4 = \square$

$= \frac{15}{100} \text{ of } \$4$

$= \frac{15}{100} \text{ of } 400\text{c}$

$= (\frac{15}{100} \times 400)\text{c}$

$= (15 \times 4)\text{c} = 60\text{c}$

$15\% \text{ of } \$4 = 60\text{c}$

The desirability of children making estimates of answers to problems has been frequently recommended in this and earlier Guides. Estimations are facilitated in many cases by rounding off. Exercises should be given to the child to develop this skill. For example :

> Rewrite the following equations using rounded numbers to obtain a result approximately equal to the result given :

(i) $3{\cdot}9 \times 11{\cdot}2 = 43{\cdot}68$ $\quad \left[4 \times 11 = 44\right]$

(ii) $17\frac{3}{4} \times 56 = 994$ $\quad * \left[\begin{array}{l} 18 \times 60 = 1080 \\ 17 \times 60 = 1020 \end{array}\right]$

(iii) $\$19{\cdot}86 \times 41 = \$814{\cdot}26$ $\quad \left[\$20 \times 40 = \$800\right]$

(iv) $\$188{\cdot}81 \div 22 = \$8{\cdot}58$ $\quad * \left[\begin{array}{l} \$190 \div 20 = \$9{\cdot}50 \\ \$180 \div 20 = \$9{\cdot}00 \end{array}\right]$

* Experience will guide the child to choose the more effective means of rounding off in order to make the better estimate.

The child should choose the most appropriate method that occurs to him to solve equations. He has had much experience in using a variety of techniques and he should be encouraged to draw on his experiences and make his own decisions in choosing the technique most suited to the problem. For example :

> Nineteen scouts went on a camp. Each contributed $5·40. Train tickets cost $2·15 per boy. Food costs amounted to $37·81. The money which remained was shared equally among the boys.

(*a*) How much did the boys' train fares amount to ?

(*b*) What was the total amount spent on fares and food ?

(*c*) If the money over was equally shared among the boys, how much did each receive ?

[N.B. This exercise might best be treated as a group project.]

(*a*) Train fares :

Estimated answer : $2 × 20 = $40 for fares

$2·15 × 19 = ☐

Possible means of solution include :

(i) $2·15 × 20 − $2·15 × 1
= ($2 + 15c) × 20 − $2·15
= $40 + 300c − $2·15
= $43 − $2·15 = $40·85
Answer : $40·85

(ii) $2·15 × 19
= $2·15 × 10 × 2 − $2·15
= $43 − $2·15
= $40·85

$$\begin{array}{l} 2{\cdot}15 \times 10 \times 2\\ = 21{\cdot}5 \times 2\\ = 43 \end{array}$$

(iii) $2·15 × 19 = $40·85

$$\begin{array}{r} 2{\cdot}15\\ 19\\ \hline 19{\cdot}35\\ 21{\cdot}50\\ \hline 40{\cdot}85 \end{array}$$

Answer : Cost of train fares is $40·85

(*b*) Cost of **fares** and food **:**

$40·85 + $37·81 = ☐

$$\begin{array}{r} 40{\cdot}85\\ 37{\cdot}81\\ \hline 78{\cdot}66 \end{array}$$

Answer : $78·66

(*c*) Refunds—

Two **approa**ches suggest **themselves :**

(i) Amount contributed by each boy minus **his** share of cost. $5·40 − $(78·66 ÷ 19) = ☐.
Estimated answer : $5·40 − $4 = $1·40

$$\begin{array}{l} \$5{\cdot}40 - \$(78{\cdot}66 \div 19)\\ = \$5{\cdot}40 - \$4{\cdot}14\\ = \$1{\cdot}26 \end{array}$$

$$\begin{array}{l} \frac{7866}{100} \times \frac{1}{19} \quad \textit{or}\\ = \frac{7866}{19} \times \frac{1}{100}\\ = 414 \times \frac{1}{100}\\ = \frac{414}{100}\\ = 4{\cdot}14. \end{array}$$

$$\begin{array}{r} 4{\cdot}14\\ 19\,\overline{)\,78{\cdot}66}\\ 76{\cdot}00\\ \hline 2{\cdot}66\\ 1{\cdot}90\\ \hline {\cdot}76\\ {\cdot}76\\ \hline {\cdot}\,{\cdot} \end{array}$$

Answer : Amount refunded to each boy = $1·26

(ii) Total surplus shared among the nineteen boys.

(Total contribution — total expenditure) ÷ 19

= (19 × $5·40 — $78·66) ÷ 19	5·40 × 19
= ($102·60 — $78·66) ÷ 19	= 5·40 × 20 — 5·40 × 1
	= 108·0 — 5·40
	= 102·60
= (34c + $21 + $2·60) ÷ 19	*or*
(this subtraction carried out by counting on)	5·40
	19
	48·60
= $23·94 ÷ 19	54·00
= 2394c ÷ 19	102·60

$$\begin{array}{r|r|l} 19 & 2394 & \\ & 1900 & 100 \\ \cline{2-2} & 494 & \\ & 380 & 20 \\ \cline{2-2} & 114 & \\ & 114 & 6 \\ \hline & & 126c \end{array}$$

Answer : 126c *or* $1·26 refund

Problems may involve ideas of ratio. Problems such as those involving comparison of prices suggested in the Guide to Section H are appropriate. For example :

(i) At a school fete comics were on sale at 2 for 15c or at 8 cents each. How many could a boy buy for $1·00 ?

(ii) Two boys attended to a football score-board for 2½ hours. Ian spent the first 1½ hours and Stuart the last hour. If the boys were paid $3 for the job, how much should each receive ?

Perhaps two boys might be given this problem to solve and then questioned to see how they reached their solution. After a number of similar exercises the approach by the pupil might be :

Ian : I worked longer than Stuart.

I worked three half-hours, Stuart worked two.

Each half-hour is worth ⅕ of $3 = 60c.

I get 3 × 60c = $1·80 ; Stuart gets 2 × 60c = $1·20.

The child should have experience with examples involving hire purchase. One such example is considered in the Guide to Section H. Details of actual hire-purchase propositions are freely available and these could be studied in discussion groups. Some investigation into hidden charges would be worth while. It should be considered that in some cases the hire-purchase system may be a valuable service (e.g. buying a house in which to live, buying a bicycle to be used in the delivery of newspapers), while in other cases a great deal of hardship may be involved to obtain an unnecessary luxury.

SPATIAL RELATIONS

SECTION H

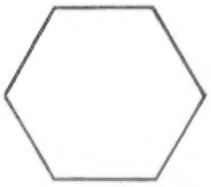
An important objective of Spatial Relations in the primary school is that children should have an awareness of simple basic shapes. This awareness should be an outcome of first-hand experience. At present the child's concern is to describe what he observes. His observation is directed to basic regular shapes and their combinations which form his complex surroundings.

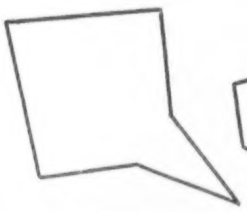
In order to describe shapes adequately a knowledge of the elements of geometrical figures has been and should be continuously developed. The child's vocabulary should become wider and more precise to enable him to cope effectively with his descriptions. (In this section the terms " obtuse ", " acute ", " north-east ", " north-west ", " south-east ", and " south-west " are introduced. Other terms he has met previously in spatial work should become more meaningful and his usage of them more precise.)

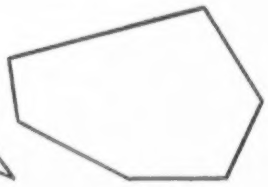
The child is familiar with many relations, for example, is on, is under, is next to, is equal to, is bigger than, is north of, and so on. Relations among elements of shapes, and between shapes, are the basis of his study. The child should be concerned with the peculiarities of a shape in isolation and with its relation to other shapes about it. It will be necessary to consider three-dimensional shapes from various points of view. One might ask, " How does the aspect of a cylindrical shape change with a change in point of view ? " or, " What shaped holes would have to be made in a sheet of three-ply to pass this cylinder through neatly if it were (*a*) standing vertically or (*b*) lying horizontally ? "

In earlier sections the child has discovered many of the relationships between shapes. He should continue to investigate triangles, quadrilaterals, pentagons, hexagons, octagons, and circles. Some classification within and between these categories can be attempted. For example :

(i) Hexagons may be regular or irregular.


REGULAR


IRREGULAR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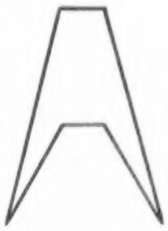
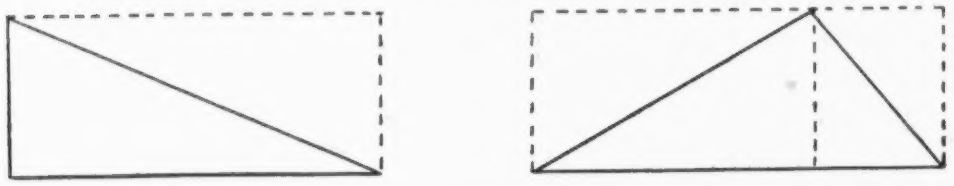
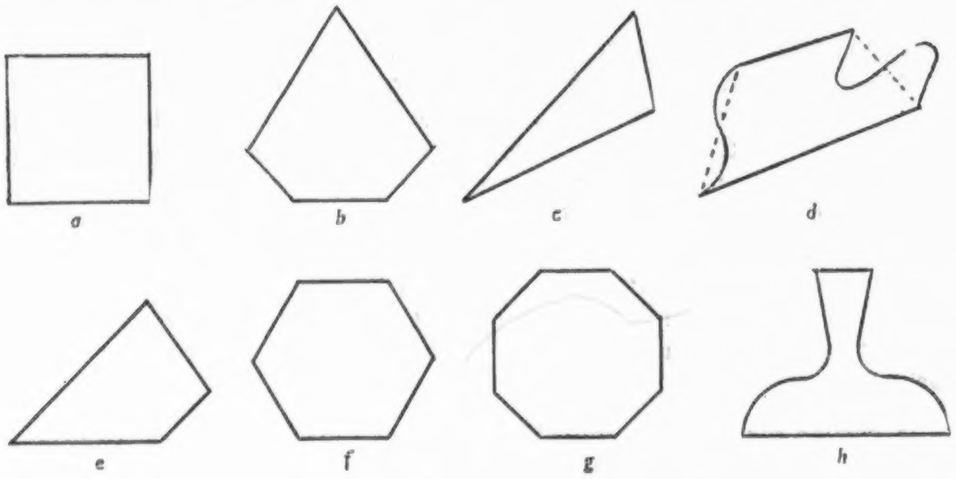

IRREGULAR

(ii) Circles, regular hexagons, and equilateral triangles are closely related.

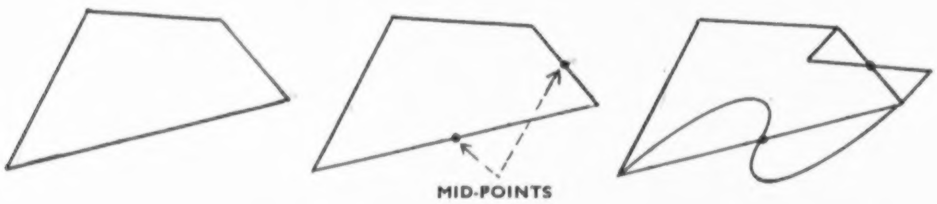
(iii) Triangles can be demonstrated as being "half rectangles".



An activity, prerequisite to an understanding of surface measurement, is that of covering surfaces with unit shapes so that no gaps are left and no overlapping occurs. The child has had some experience in this activity which can be called paving or tessellating. If multiple copies of different shapes (say, two shapes at a time) are given to the child, he can discover which shapes will and which shapes will not tessellate. (Children may prefer colored papers that form patterns when the shapes are pasted on paper.) The following diagram shows some shapes which will tessellate (*a*, *c*, *d*, e, and *f*) and some shapes which will not tessellate.



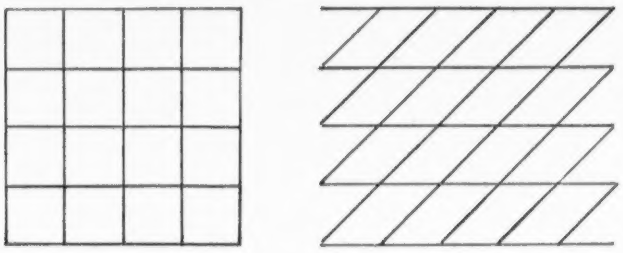
Shapes, such as *d* above, can easily be prepared by taking a quadrilateral, marking a central point on one or more of the boundaries, and then constructing symmetrical curves about the central point and joining the vertices. The following sequence illustrates this :



After having had some experience with this activity, the child might be presented with one cut-out of a particular shape, say a regular octagon, and challenged to say whether the shape will or will not tessellate. Again he might be asked to suggest why shape *d* will tessellate and why shape *h* will not tessellate.

Patterns in dress materials, wall-papers, mosaic tile patterns, linoleums, and such like might be examined in order to discover if a tessellating shape has been used in constructing the pattern.

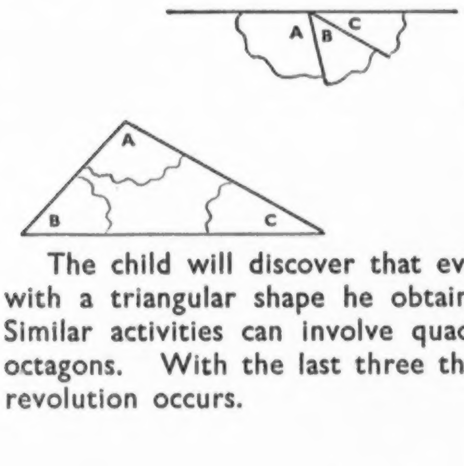
Simple tessellating shapes are used to make up graticules which children should use to develop into patterns of their own design. Compasses, rulers, and set squares can be used in these activities. As an outcome of this work, relationships between various two-dimensional figures may become evident to the child. The child may recognise the relationships between circles, hexagons, and equilateral triangles. He may also discover the difficulty of using both equilateral and right-angled triangles on the one graticule.

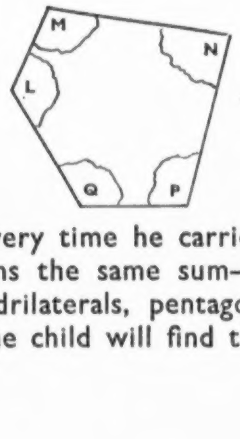


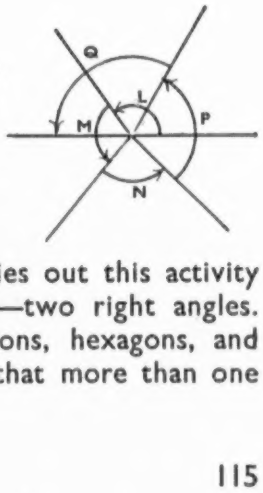
Simple graticules

The angles at the vertices of plane figures may be considered. The right angle will serve as a suitable means of comparison. A model for this angle was suggested in Section G. This may still be used although a set square (45°) is a useful instrument. The vocabulary terms " obtuse " (an angle of magnitude between one and two right angles) and " acute " (an angle of magnitude between zero and one right angle) should be introduced. Angles may now be described, in terms of magnitude, as turns or revolutions, right angles, half-right angles, acute angles, or obtuse angles. Triangles may be described as having three acute angles, a right angle and two acute angles, a right angle and two half-right angles, an obtuse angle and two acute angles, and so on.

The magnitudes of angles can be added. To find the sum of the magnitudes of the angles of a plane shape, the child can take a paper model of the shape, tear off the corners, and rearrange them as shown below.

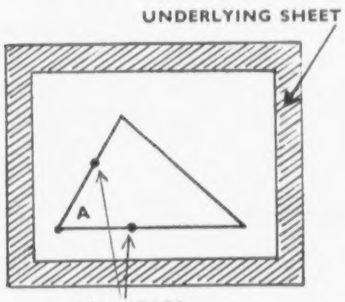




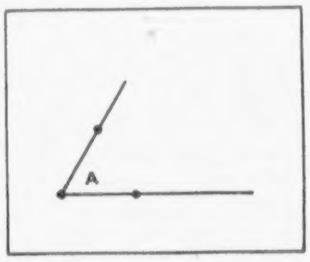


The child will discover that every time he carries out this activity with a triangular shape he obtains the same sum—two right angles. Similar activities can involve quadrilaterals, pentagons, hexagons, and octagons. With the last three the child will find that more than one revolution occurs.

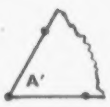
Angles can be directly compared by taking a paper model of one of the angles and superpositioning it over other angles that are to be compared with it. To make such a model one would need only to prick through to an underlying paper and cut out the model as in the diagram below.



PIN-HOLES



LINES THROUGH PIN-HOLES



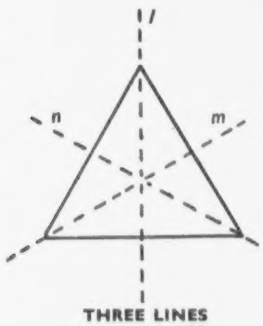
MODEL

From comparisons of angles formed by boundaries of figures, from the sums of the magnitudes of the angles, and from a consideration of the relative lengths of the boundaries, many properties of rectilinear figures can be discovered by the child. For example :

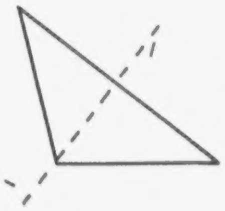
(i) The sum of the magnitudes of the angles of any quadrilateral is four right angles.

(ii) If a triangle has three "equal" sides it also has three "equal" angles. The word *congruent* is sometimes used instead of equal.

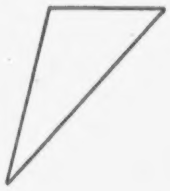
Plane and three-dimensional shapes can be described in terms of symmetry or balance. The child was first introduced to symmetry in Section D. Since that time he has had experiences developing the appreciation of bilateral symmetry or balance about a line. He has examined simple geometrical shapes and may have come to appreciate that some figures have more than one line of balance. An equilateral triangle may be described as having three lines of balance ; an isosceles triangle as having one such line. (The terms "equilateral" and "isosceles" are not required of the child.)



THREE LINES OF BALANCE



ONE LINE OF BALANCE



NO LINE OF BALANCE

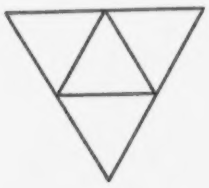
Some discussion in terms of lines of balance can be taken in regard to three-dimensional objects. Consider, for example, motor-cars, furniture, ice-cream cones, cylindrical tins, other containers.

The child should have much experience with three-dimensional shapes. He should be familiar with triangular and square pyramids and prisms, right circular cylinders, cones, spheres, and sectors of spheres. A valuable reference is *Background in Mathematics*, published by the Education Department of Victoria and the Australian Council for Educational Research, 1966, p. 202 ff.

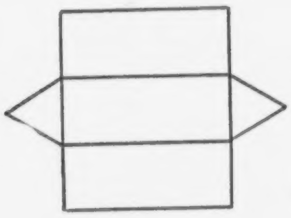
Facilities should be made available and exercises planned so that the child will make solid models of cones, cylinders, spheres, and so on, from clay, plasticene, or other such material. The solids may be cut and examined in cross section or threaded onto cotton through a line of balance. When spun on such an axis the model should not appear to change its aspect.

Models of three-dimensional figures may be made from materials such as drinking straws joined by pipe-cleaners or from matches and glue.

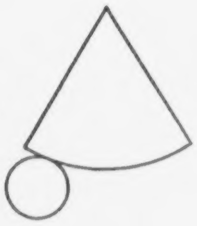
Some three-dimensional shapes can be made by paper-folding and pasting. If a shape can be totally covered by a sheet of paper without the paper crumpling, it is said to be developable. Thus rectangular and triangular prisms and pyramids, cones, and cylinders are developable, while a sphere, or part of a sphere, is said to be non-developable. The two-dimensional shape that can be folded to form a three-dimensional shape is said to be the net or development of the three-dimensional shape. The child should endeavor to find the nets for himself. He may do this by observation and reflection ; he may need to wrap the shape and mark the wrapping appropriately ; he may unfold a ready-made article such as a matchbox or a milk carton. A familiar illustration of a development is the paper label on a food tin. Some simple nets are illustrated below.



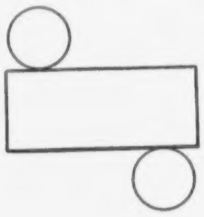
TETRAHEDRON
(TRIANGULAR PYRAMID)



TRIANGULAR
PRISM

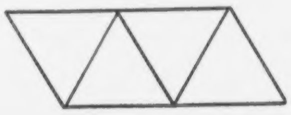


CONE



CYLINDER

The above illustrations do not represent the only arrangement, nor are they the only way in which the nets may be made. Consider the tetrahedron. Other arrangements for this net may be :



OR

In the second case, after folding this rectangle into a cylinder and then crimping the upper and the lower edges at right angles to one another, a tetrahedron may be formed by making appropriate creases in the curved surface. This method is used in the manufacture of drink (e.g. milk) cartons. Various containers should be examined in an endeavor to discover the nets used for their construction.

Every opportunity should be taken to investigate the properties of various shapes which fit them for particular functions in industry, architecture, engineering, and nature. Properties such as rigidity, strength, elegance, ease of manufacture, ease and stability in stacking, or holding capacity may be discovered. One might well consider why wheat silos, petrol tankers, water pipes, oil drums, preserved-food tins, telephone poles, or pencils are basically cylindrical. Again, one might wonder why, in some areas, milk cartons have replaced milk bottles. The shape of the Australian Rules football will provide a topic for lively discussion.

In Section G the child learned to orientate himself in the schoolground in terms of the cardinal points of the compass. Now he can extend this notion to include north-east, north-west, south-east, and south-west. He should continue to give accurate, simple directions based on landmarks and left/right directions, or on the eight points of the compass. Some activities should involve grids and number pairs to locate positions. Practice in locating positions in street-directories should be given. The popular usage of the combination of letters and numerals (for example, G8) to designate location should receive consideration.

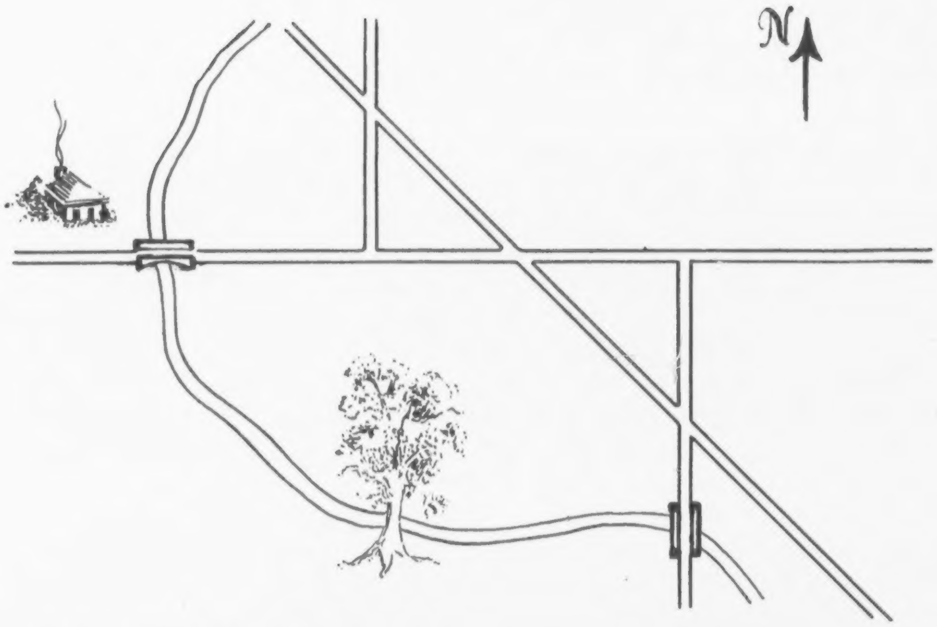
The child may be familiar with reference grids such as are commonly used in theatres. In the classroom one might say "the third desk from the front in the second row from the door". One speaks of columns and rows on a number board. Given a rectangular reference grid, the child might be asked to name the numeral written in the fourth row of the third column. (Columns and rows would, of necessity, require definition, say, vertical columns and horizontal rows.) When the child can locate points in a plane using the idea of rows and columns, or similar description, he may symbolise, say, rows by letters of the alphabet and columns with numerals. A child may wish to use his own system for labelling rows and columns.

Many games can be devised (by children and/or the teacher) in which co-ordinates are used, for example, a treasure hunt. Using a plan of the school room (or other suitable area) a rectangular grid is superpositioned and a reference system is devised. Children can then be directed to points in the room where clues may be discovered. Again, using reference points, hidden pictures can be discovered and drawn. The child should be given opportunities to create problems for his classmates to solve. In some practical situations more informal references will be necessary. Natural objects (e.g. trees) and places may be suitable. Directions for locating positions would then need to be given in words. Experience will show that at least two references are needed to avoid ambiguity and thus pin-point a position. The child will also discover that, having located a point on the map, he will have to line up prominent features shown on the map in order to locate the point in the actual situation, since there is no rectangular grid set down on the ground.

The child should have experience both in reading and in preparing simple maps. He should show development by the complexity of directions that he can follow. Such complexity should not, however, be developed prematurely. Initial exercises will, of necessity, be very simple. Maps will be of the classroom, the school-ground, the boy's home, or some very familiar area. The teacher may prepare a plan of the classroom and ask the child to point out various features, for example, his own seat. A boy might prepare a plan of a football oval and indicate the various positions on it. He can indicate the northerly direction and show the location of some adjacent buildings.

The child might be given a fictitious map and a set of directions, and then be asked to follow the directions and name his destination. He may be given an exercise such as the following :

> A boy is walking along a road (shown in the diagram on page 120) in an easterly direction. He crosses a bridge and soon arrives at a cross-road. He walks on a little further, without changing direction, and arrives at another road. He then turns to the right. At the next cross-road he follows a road to his left.
>
> (i) In which direction is he now facing ?
>
> (ii) Using the points of the compass describe his shortest route, by road, to the bridge near the house shown in the map.



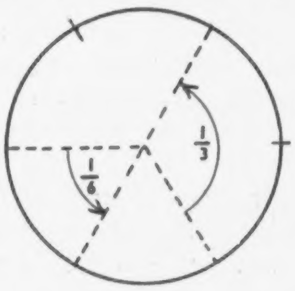
It will probably be found that frequent periods of short duration (say ten minutes, twice a week), in which problems are presented to the child for his consideration and investigation in his free time, will more satisfactorily develop the objectives of Spatial Relations than will formal class lessons taken once in three or four weeks. This is not to say that group and/or class discussion should not occur. On the contrary, such interaction involving questions and the contribution of ideas is very important.

SECTION I

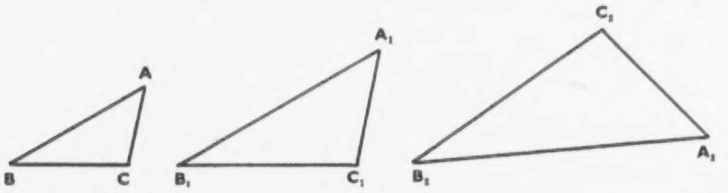
Topics considered in Section H should be maintained and further developed in Section I.

Further exploration of the properties of two-dimensional shapes should be carried out.

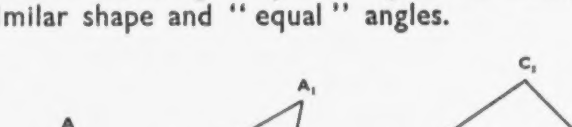
Angles can be described in terms as set out in Section H. In addition $\frac{1}{3}$, $\frac{1}{6}$, and $\frac{1}{8}$ turns can be used. Models for $\frac{1}{3}$ and $\frac{1}{6}$ turns can be obtained from the circle. If the child has not discovered how he can partition the circle into six "equal" parts, he should now do so. The models for $\frac{1}{3}$ turn and $\frac{1}{6}$ turn can be cut from thin card. A model for $\frac{1}{8}$ turn can be obtained by halving the right angle.



As an outcome of his study of angles, the child will discover that shapes which are similar—that is, those that are of the same shape but possibly scaled up or down in overall size—have matched pairs of angles equal in magnitude. Activities in which the child sorts models of triangles in terms of matching " equal " angles will assist him to relate the ideas of similar shape and " equal " angles.



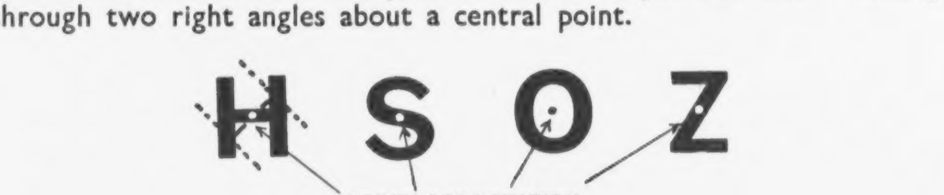
More specific attention can be directed to symmetry as observed in two-dimensional and three-dimensional shapes. The two main types of symmetry can be observed in letters of the alphabet. Rotational symmetry—symmetry about a point—can be seen in H, I, N, O, S, X, and Z. A point exists such that the letter can be rotated less than one full turn and yet appear as it did before rotation. Each of the letters above returns to its present form and position after rotating through two right angles about a central point.



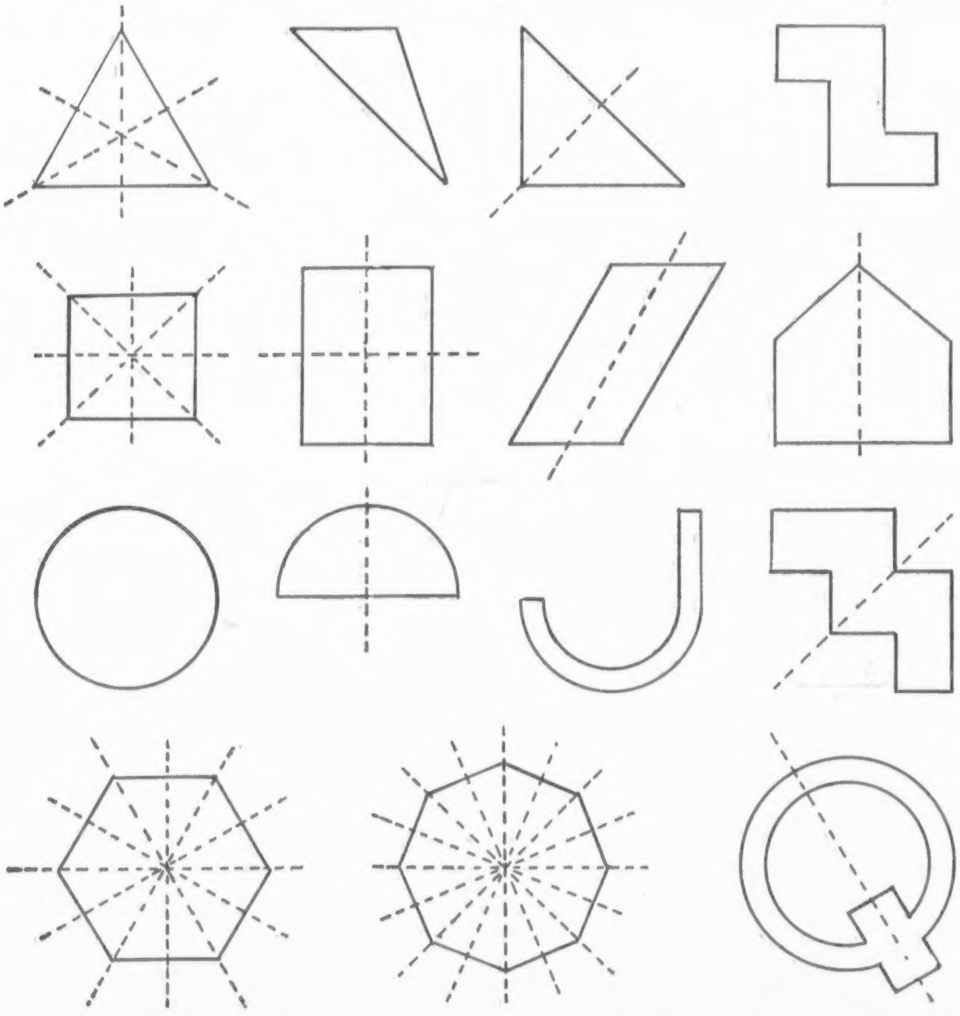
During a complete rotation an equilateral triangle takes up three identical positions, and a square four such positions. The child may be given a number of two-dimensional shapes and challenged to find points of symmetry (where they exist). In order to accomplish his task he might outline a shape and then turn the shape until it fills its previous outline. To determine the point of rotation the child might take a pin and make trials ; some children may employ rulers and pencils to assist them.

A reconsideration of shapes used in tessellation (see Spatial Relations, shapes *d* and *h*, page 114) would be a worth-while study of an application of point symmetry.

In earlier sections the child has been mainly concerned with bilateral symmetry. He has seen balance in shapes about him—in faces, butterflies, leaves, motor-cars, footballs, and geometric shapes. He has possibly investigated symmetry in letters of the alphabet. For example

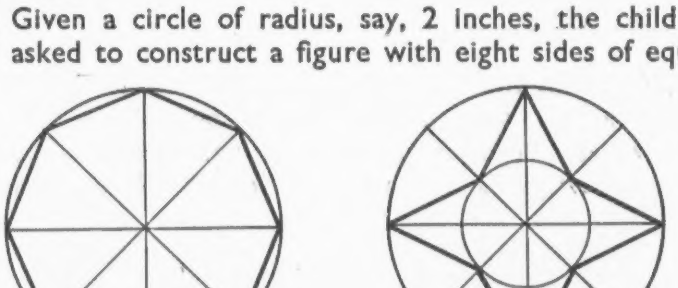


The child can examine shapes such as the following and mark in the lines of symmetry (where they exist).

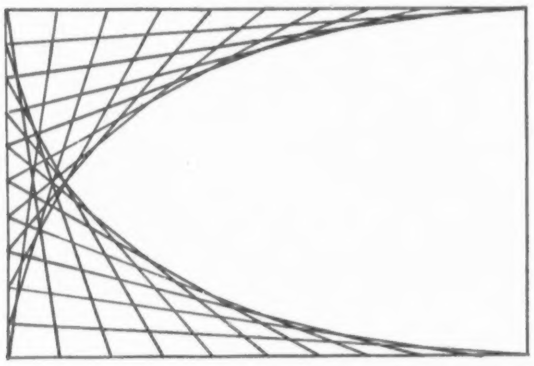


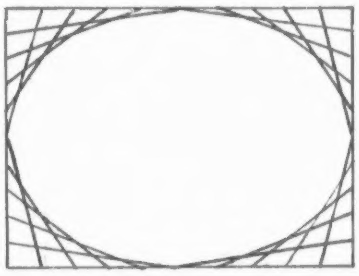
The child has developed a measure of skill in ruling and measuring activities. This development can be utilised and furthered in pattern work based on ruled graticules, as in Section H, and in activities such as the following :

(i) Given a circle of radius, say, 2 inches, the child might be asked to construct a figure with eight sides of equal length.



(ii) The child constructs a rectangle, using his ruler and set square. (This activity will itself need practice before the final result satisfies the child.) Each boundary of the rectangle is marked to have a given number of equal intervals. Using some predetermined sequence of his own choosing, the child rules lines to join dots on adjacent sides of the rectangle.





Similar pattern work can be done with other shapes. The child should be encouraged to experiment for himself.

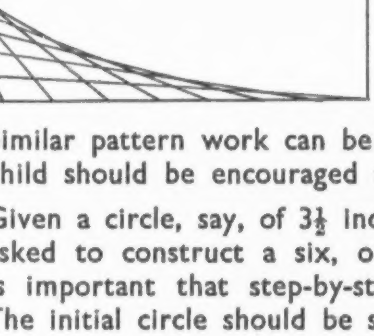
(iii) Given a circle, say, of 3½ inches radius, the child might be asked to construct a six, or a twelve, pointed star. (It is important that step-by-step instructions be not given. The initial circle should be sufficient.)

In Section H, the child had experiences involving the nets or developments of simple three-dimensional shapes. In Section I the nets of more complex figures may be investigated and the nets of simple figures may be reconsidered in different arrangements. Some exercises are suggested below :

(i) Given a funnel, the child is challenged to make a similar one from thin card and cellulose tape.

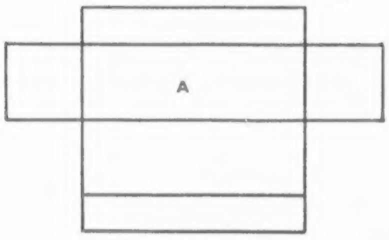
(ii) Given the dimensions of a school textbook familiar to him (say, his reader), the child is required to make a paper cover for the book without handling the book.

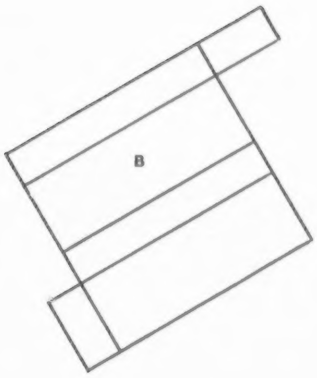
(iii) Describe the shape which has a net as shown.

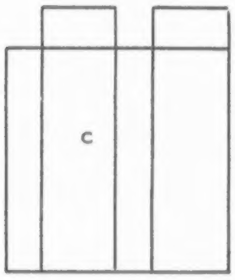


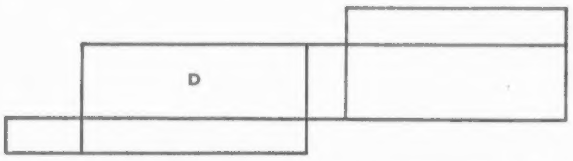
SHAPE IS A PLASTIC
OIL-CAN WITH AN
OVAL BASE

(iv) Which of the following nets can be folded to form a closed rectangular box ?



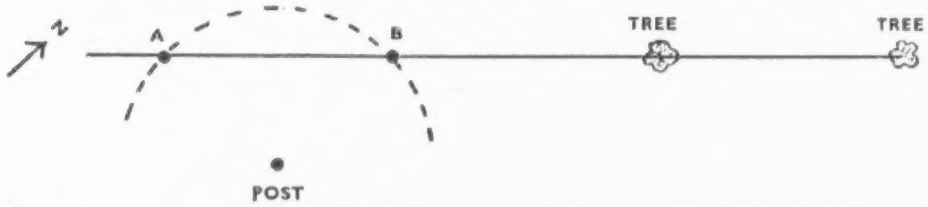






In Section H attention was drawn to the need to consider the shape of an object in relation to its function. This investigation of the properties of various shapes which influence designers and others in choosing shapes should continue. The consideration of the development of the motor-car body in terms of shape would be a useful project. Shape should be considered in more than aesthetic terms.

The preparation of maps and the reading of prepared maps should continue. Rectangular reference grids, as suggested for Section H, and some other simple references involving the intersection of straight lines (for example as in direct navigation) or of circles and lines should be frequently employed. The location of a point may be described as " in line with two trees and at a given distance from a post ".



It is seen that two positions are possible. Some further information is required, say, that the post is south of the point. The child might be asked to indicate what unnecessary information was given. (In this case the distance from the post to the point in question.)

Exercises, such as the one suggested in the Curriculum Guide, Section H, pages 118—120, should be employed to give the child practice in following directions and in map reading. The child should also prepare exercises for others to follow. Road maps, street directories, atlas maps, fictitious maps, plans, and blue prints can all be used. Some location exercises can involve the use of aerial photographs of a familiar locality, if such photographs are readily obtainable. The experiences of boy scouts in map-making and map reading may well be used to advantage in group work in this topic.

STATISTICS AND GRAPHS

SECTION H

In Section G the child worked with pictorial graphs and bar graphs. Construction and interpretation of graphs were emphasised. Activities were based on the child's experiences in his environment.

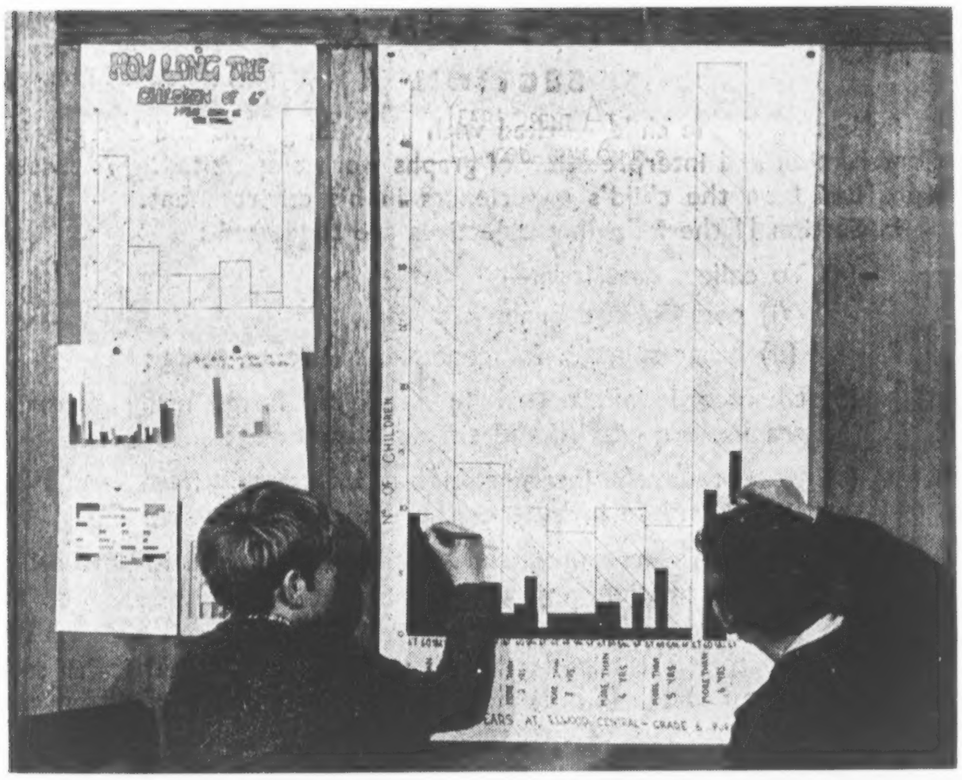
In Section H the following objectives are suggested :

- (*a*) To collect data from—
  - (i) personal and group activities,
  - (ii) other sources, for example, reference texts ;
- (*b*) to record information in graphical form, using pictorial graphs, bar graphs, and simple histograms ;
- (*c*) to continue the interpretation of information from graphs (as above) and to investigate the use of line graphs ;
- (*d*)
  - (i) to understand when a graph can be useful in portraying information, and
  - (ii) to make the most suitable choice of graph type to do this (the types of graph used to portray particular sets of data will depend on the data itself and on the outcomes of discussions involving teacher and children) ;
- (*e*) to carry out simple investigations of random events and so discover any long-term regularity in frequencies of events.

While graph paper is not essential, its use will allow the child to focus his attention on the construction of the graph itself without the tedious mechanical preparation of grids. Some form of prepared paper is very helpful to the child. This can be either a commercially available paper (¼-in. grid) or paper with grids duplicated for the purpose.

The terms "frequency" (the number of times an event occurs) and "tally" or "tallying" (the record of the frequency) may profitably be introduced. For example, the record of a traffic count may be kept as follows :

Event	Tally	Frequency
(Vehicles passing)		
Cars	~~IIII~~ ~~IIII~~ III	13
Trucks	III	3
Motor-cycles	I	1
Bicycles	IIII	4



The method shown on page 125 for tallying is often used. It enables events to be recorded and totals to be made efficiently.

Comments made in the Guide to Section G, Statistics and Graphs, in relation to the preparation and the interpretation of pictorial graphs and bar graphs are relevant in Section H.

Science and social studies topics will provide useful material for graphing as well as strong motivation for graph work.

The Histogram

The histogram is a special form of bar graph. It is suitable for use when the data to be graphed is of a continuous nature. For example, in a graph showing the weights of children in a classroom, one axis will show numbers of children and the other will show a scale of weights. The "number of children" axis will be marked in whole units and no fractions are possible. The "weight" axis will represent a continuous scale. Every point on this scale represents a weight, even though only a few will be marked. (The children represent discrete quantities where whole numbers only are possible ; the weight scale represents a continuous quantity ; fractions do occur.)

The following topics are suitable for the portrayal of data using histograms :

(i) Weights of children (see Guide to Section G, pages 111, 112).
(ii) Heights of children.
(iii) Ages of children.

(iv) Temperatures at midday over one month.

(v) Times taken by children to run 880 yards.

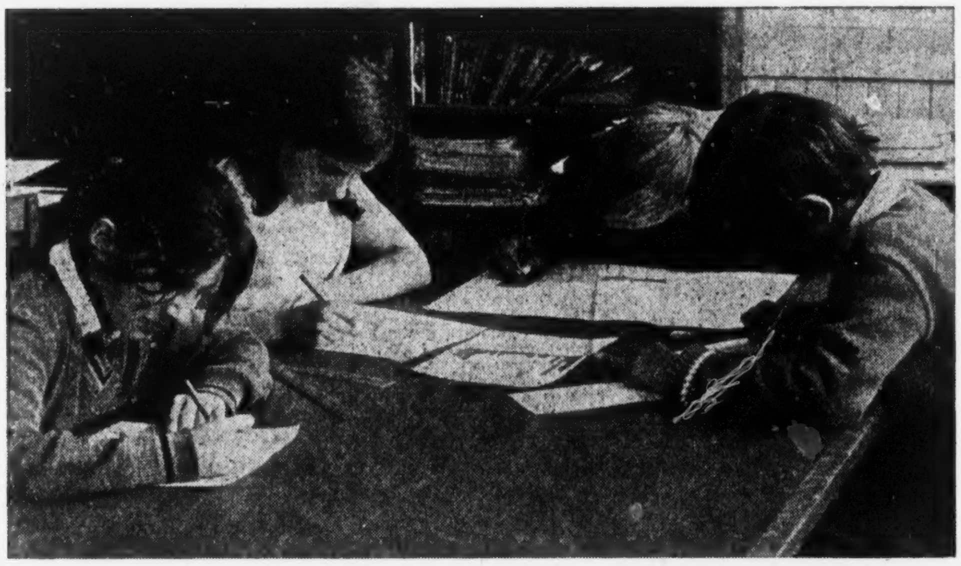
The following data might have been collected from a class activity in which the time was taken for each child to run 880 yards :

182 sec. 173 sec. 223 sec. 268 sec. 265 sec. 203 sec. 261 sec.
161 sec. 131 sec. 152 sec. 149 sec. 194 sec. 243 sec. 211 sec.
168 sec. 198 sec. 182 sec. 223 sec. 149 sec. 249 sec. 231 sec.
194 sec. 152 sec. 131 sec. 168 sec. 152 sec. 211 sec. 255 sec.
182 sec. 173 sec. 135 sec. 203 sec. 173 sec. 182 sec. 223 sec.
223 sec. 207 sec. 211 sec. 231 sec. 182 sec. 207 sec. 194 sec.
231 sec. 194 sec. 161 sec. 240 sec. 194 sec.

As it stands, this data is difficult to comprehend. In order to simplify the presentation the times might be rearranged as follows :

TIME (in seconds)	TALLY	NUMBER OF CASES (frequency)
131	II	2
135	I	1
149	II	2
152	III	3
161	II	2
168	II	2
173	III	3
182	𝍸	5
194	𝍸	5
198	I	1
203	II	2
207	II	2
211	III	3
223	IIII	4
231	III	3
240	I	1
243	I	1
249	I	1
255	I	1
261	I	1
265	I	1
268	I	1
		Total 47

A class (or group) discussion on the ways this data can be graphed might now take place. It is possible to construct a bar or a column graph directly from the data. This would require 22 bars or columns, each a fine line. (Why a fine line ?) Perhaps the data could be graphed so that various levels of achievement are shown, for example,



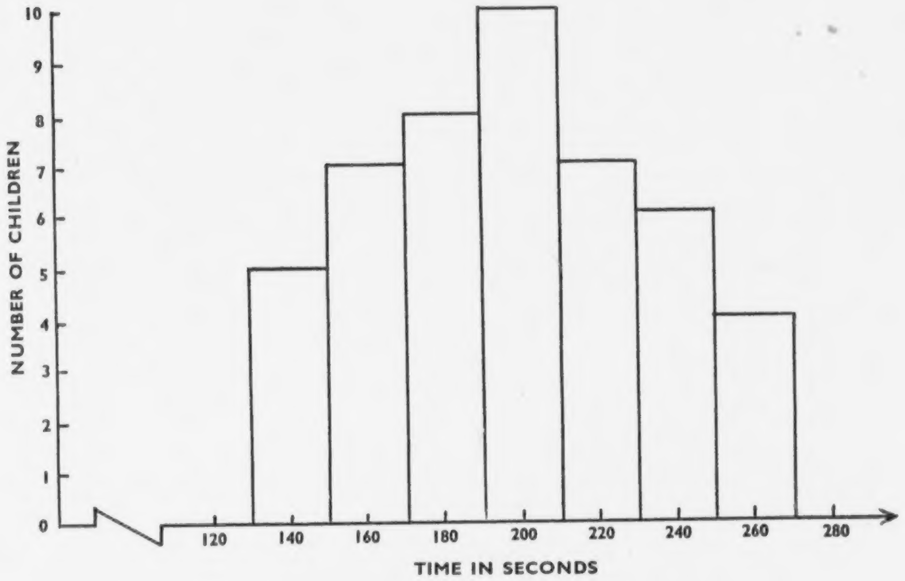
to show how many children take less than 130 sec., 150 sec., 170 sec., 190 sec., 210 sec., 230 sec., 250 sec., and 270 sec., to cover the distance. To do this it would be necessary to divide the times into intervals such as (110 to 129) sec., (130 to 149) sec., and so on. It should be understood that such intervals are mutually exclusive, that is, no case can fall into two categories at one time as might be the case if the intervals were stated as 110 to 130, 130 to 150, and so on.

Organisation of the data to suit these levels may result in the following table :

Interval (seconds)	Number of Cases
110 to 129	0
130 to 149	5
150 to 169	7
170 to 189	8
190 to 209	10
210 to 229	7
230 to 249	6
250 to 269	4
	Total 47

The question of how the time axis was to be marked would also have to be considered. It could be done by marking and labelling the points 110, 130, 150, . . . 250, 270. Alternatively, it could be done by marking the points 110, 120, 130, 140 . . . 260, 270 and labelling the alternate points. (120, 140, 160, . . . 240, 260, 280.) The group might even consider some other method to be more suitable.

TIME TAKEN BY CHILDREN TO RUN 880 YARDS



Discussion might centre on questions such as :

(i) How many children ran the distance in less than 170 sec. ?

(ii) In which time interval are most children found ?

(iii) Which two time intervals have equal numbers of children ?

(iv) How many children took 190 or more seconds to run the distance ?

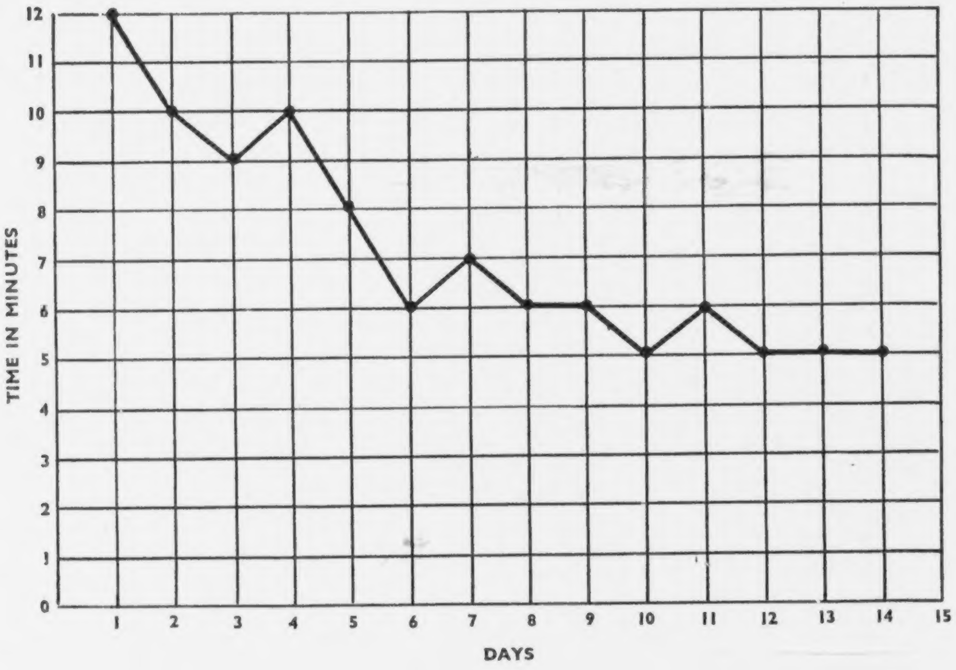
(v) Can you suggest another way in which the results of the running can be shown more suitably ? If so, describe the method.

Line Graphs

The child is introduced to the notion and use of co-ordinates in Spatial Relations, Section H (see Curriculum Guide, Section H, pages 118—120). The child's work in line graphs will develop from his understanding of co-ordinate systems.

One way to introduce the line graph would be to supply the child with a prepared paper showing a quarter-inch rectangular grid with the two axes marked appropriately (see diagram on page 130) and a title indicating the activity concerned. On each occasion the child completes the given task, he makes the appropriate entry on the prepared sheet. Successive entries are linked with lines. The lines make it easier to see how performance alters.

TIME TAKEN EACH DAY TO COMPLETE A JIGSAW PUZZLE



The child can interpret this graph as he would a bar or column graph. The highest points of the columns are joined so that trends are more easily seen. This line graph is useful when there are many points to be plotted. Discussion could centre on questions such as :

(i) On which day was the most time taken ?

(ii) On how many occasions did the child take six minutes to complete the task ?

(iii) What was the shortest time taken ?

(iv) How many times was the task completed ?

Similar graphs could be drawn to show the temperature at a given time each day ; the length of a shadow at midday on successive days ; a child's weight at a given time each week ; the saving a child effects from his pocket money each week ; the number of papers sold each day ; and so on.

Investigation of Random Events

The child should discover that the long-term trend in a frequency of events can be predicted, but that it is not possible to predict a single occurrence.

In certain circumstances the long-term trend suggests a probability of one in two that an event will occur. Thus the probability of a baby being a boy is one in two, that of a coin landing heads (or tails) is also one in two. These results become evident if a large number of cases is considered.

In other circumstances the probability of an event occurring may be of different order, say, one is to three, or one is to four, and so on. Thus the child may make a traffic count and discover that certain makes of cars pass more frequently than others. He may find that he can guess correctly, say one time in four, that the next car will be one of a particular popular make.

The outcome of a number of throws of two coins (or of dice or of similar objects) is interesting. A regular pattern becomes discernible.

A useful approach to this topic is set out in *Background in Mathematics*, published by the Education Department of Victoria and the Australian Council for Educational Research, 1960, Probability and Statistics, pages 173–176. The depth of treatment for this topic should depend on the background and the ability of the individual child.

SECTION I

Activities suggested for Section H may be continued in this section. The child will show an increasing independence of the teacher and of prepared material by choosing his own scales, class intervals, and form of representation. He should show greater facility in reading off information from prepared graphs found in texts (mathematical, science, social studies, etc.), charts, and other sources.

Some aspects of the child's experience with co-ordinates have involved the location of points suitably designated by intersecting lines, or points located in areas distinguishable by grid references. He should continue to have experience in locating points from grid references. The child can prepare his own reference system and name points on it for his neighbor or other class members to find.

A datum or starting-point should be selected. Street-directories generally use the top left-hand corner of the diagram for this purpose. Many graphs are constructed on the basis of the lower left-hand corner being the datum. The axes intersect at this point and readings are given from it.

In his early attempts, the child may need to describe the chosen point in terms such as four across and two up. Later he may develop a notation such as (4, 2) to mean the same thing. It will become clear that this understanding must be shared by other members of his class or confusion will develop.

Why should such a convention be chosen ? The child will discover that most directories and atlases name the horizontal reference first in a pair of co-ordinates. It is probably desirable to adopt this convention since it is the one most commonly used in texts containing graphs.

A useful discussion of co-ordinate systems may be found in *Background in Mathematics* (mentioned above), pages 218, 220–222. At this time activities should be restricted to those involving only the positive whole numbers and zero.

Arithmetic Mean, Mode, and Median

The following activity is valuable in developing an understanding of the terms "arithmetic mean", "mode", and "median". Each child covers a paper square with small leaves. The paper squares are of the same size and the leaves are of the same shape and size. Leaves should not overlap and should cover as much of the paper as possible. Each child counts the number of leaves he has used. The following table may result from recording the numbers :

Number of Leaves	Number of Cases (f)	Leaves Used
18	1	18
19	1	19
20	2	40
21	2	42
22	4	88
23	4	92
24	4	96
25	1	25
26	0	0
27	1	27
28	1	28
29	5	145
30	3	90
31	4	124
32	4	128
	Total 37	Total 962

From a discussion of the results organised as above the following points can emerge :

1. The number of leaves used ranged between 18 and 32. (An examination of the sheets may explain why so many different results were obtained.)
2. Thirty-seven children carried out the exercise.
3. Nine hundred and sixty-two leaves were used.
4. If all children had used the same number of leaves and 962 leaves were used, each child would have used $\frac{962}{37}$ or 26 leaves. This number of leaves is called the *arithmetic mean*, the *mean*, or sometimes the *average* number.
5. The point on the scale which has the same number of cases above it as below it is at 25 leaves. This figure (25) is often called the *median* in the distribution of cases.
6. The point on the scale which has the greatest number of cases is called the *mode*. In this situation the point is 29.

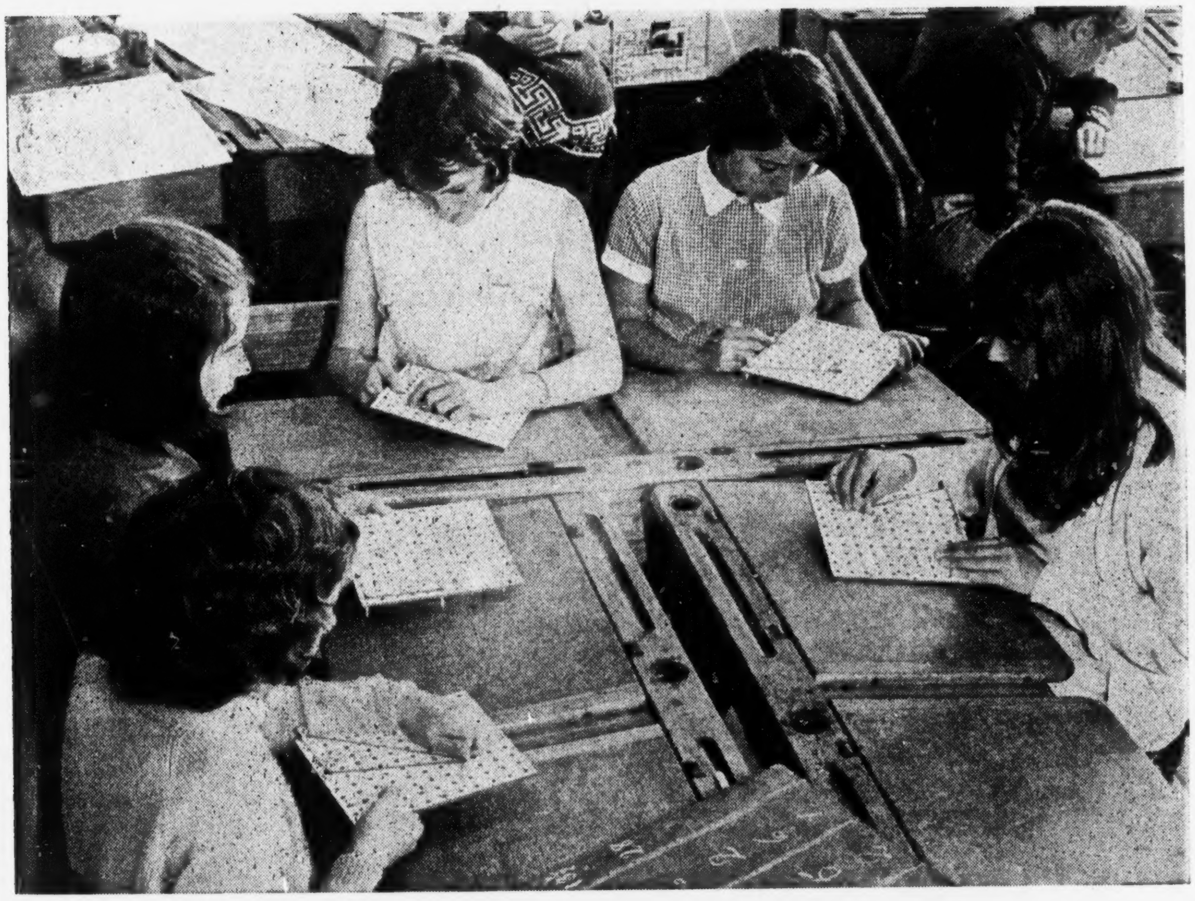
(In some distributions all three of these measures, (mode, median, mean) coincide. In others the median and mean coincide. Sometimes more than one mode may occur—this would have occurred in the example above if, say, there were 5 cases of 23 leaves and 5 of 29 leaves.)

The child should consider which is the most useful of these measures for particular situations. Which measure is most important to the child when the subject of pocket money is discussed ? (The kind of distribution is very important here, as it is in most discussions of this nature. Consider two possibilities :

(i) A small top group received much money, the remainder very little by comparison.

(ii) A very large proportion of the group received a similar amount of money, with no marked increase at the top of the range.)

A useful discussion of these measures is to be found in *Background in Mathematics*, pages 179–182.



GLOSSARY

SECTIONS G, H, I

This glossary supplements that given in the Curriculum Guide, Pure Number, Section F, pp. 109–117. Definitions supplied do not necessarily take into account all applications of the term defined. The intention is to define the given term so that its use in the text is clearly understood.

Abacus—A device for calculating, usually involving the sliding of beads or counters along a wire.

Acute Angle—An angle of magnitude less than one right angle.

Adjacent Angles—Two angles having a common arm and a common vertex and lying on opposite sides of the common arm.

Aliquot Part—Any exact divisor of a quantity ; any factor of a quantity—used almost entirely when dealing with whole numbers.

Angle—A basic figure in geometry made up of two half-lines or rays with a common starting-point. The rays are the *arms* of the angle and their common starting-point is the vertex.

Approximation—A result that is nearly but not exactly correct, but which is accurate enough for some specific purpose.

Area—The numerical measure in terms of a specified unit, which can be given to a plane region.

Arithmetic Mean—A measure of the location of the centre of a distribution. For simplicity, the arithmetic mean value is frequently called the *mean* value, or sometimes the *average*.

Array—An orderly grouping or arrangement of things, marks, or numerals in rows and columns.

Base (of a Place-value System of Numeration)—The number used in the fundamental grouping procedure ; e.g. 10 is the base in the decimal system, 2 is the base in the binary system.

Callipers—Compasses with bowed legs, turned inwards or outwards, to make external or internal measurements.

Centimetre—One-hundredth part of a metre.

Circumference—The boundary of a circular region. Circumference usually implies the length measure of the boundary.

Common Denominator—A number which is divisible by all the denominators under consideration.

Common Divisor—A number that is a factor of each of two or more numbers.

Complementary Addition—A means for solution of equations involving subtraction; e.g. $7 - 4 = \square$ can be rewritten as $\square + 4 = 7$ and solved through the knowledge of an addition fact.

Compound Quantity—The magnitude of a given quantity expressed in two or more units, e.g. 3 ft. 6 in., 3 tons 15 cwt. 110 lb.

Cone—A cone is a three-dimensional shape with a plane base bounded by a circle or other simple closed curve. It has a vertex point which does not lie in the plane of the base. A curved surface joins the vertex to all points on the closed curve. Any line which passes through the vertex and any point on the closed curve is a "straight" line.

Congruent Shapes—The term "congruent" in this context expresses the idea that two or more figures or regions have the same shape and size but may have different locations and orientations.

Conservation—The principle of:

- (i) for number—the cardinal number of a collection remains unchanged when its elements are rearranged;
- (ii) for length—the length of a line segment is unchanged when it is bent or broken into parts and rearranged;
- (iii) for area—the area of a plane shape is unchanged when the shape is partitioned and rearranged;
- (iv) for volume—
  - (*a*) the volume of a three-dimensional shape remains unchanged when the shape is partitioned and rearranged;
  - (*b*) the volume of a fluid is unchanged when the fluid is poured into different vessels;
- (v) for weight—the weight of a substance is unchanged when the shape is changed or when it is partitioned and the weights of the individual parts are summed.

Continuous Quantity—A quantity that can be measured in fractions of a unit; for example, in length or in weight. Between any two measures, no matter how close, another measure can be found. Discrete materials cannot be measured in fractions; counting numbers only are appropriate; for example, bicycles, boys, buttons.

Continuous Scale—A scale used to measure a continuous quantity; that is, it allows for measures of units and fractions of a unit.

Co-ordinates—The ordered pair of numbers associated with a point in a plane are called the co-ordinates of the point, with reference to given co-ordinate axes.

Cubit—An ancient unit for length measurement which corresponded to the length of the forearm, (about 20 in.)

Curve—An undefined term used in geometry to express the idea of a path which may turn in any manner as it proceeds indefinitely in either direction. A line is a particular curve that does not change direction. (The term " line " used strictly refers to a curve which does not change direction—" straight line " and " line " have the same meaning.)

Cylinder—A cylinder is a three-dimensional figure that has bases that are simple closed curves lying in parallel planes. The lateral surface (the curved surface) is such that parallel line segments joining corresponding points on the bases lie in the surface.

Data—Information supplied.

Decade Addition—Addition by tens.

Decimal Fraction—A numeral in base ten representing a rational number.

Decimal Notation—A fraction written in decimal form. For example, $\frac{3}{4}$ written in decimal form is 0·75 ; similarly, $7\frac{1}{4}$ is rewritten as 7·25. The numeral 174 is already expressed in decimal form if it is assumed to be in base ten.

Decomposition—Renaming involving separation into constituent parts; for example 6 yards 2 feet might be renamed 5 yards 5 feet.

Diagonal—A line segment that connects two non-adjacent vertices of a polygon.

Diameter (of a Circle)—A line segment whose end points lie on the circle, and which contains the centre of the circle.

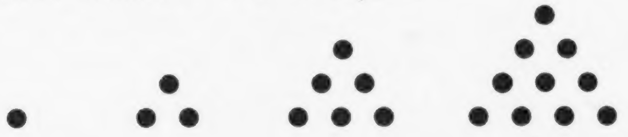
Dimension—A measurable attribute such as length, weight, area, volume. (The *three dimensions* usually refer to length, width, and height.)

Discrete Materials—(See **Continuous Quantity**). Individually distinct objects.

Dot Arrays—An orderly grouping or arrangement of dots—usually rectangular with clearly defined rows and columns.

Equilateral—Having sides with equal measures of length—usually used in the context " equilateral triangles ".

Figurate Number—Geometrical patterns, usually of dots, representing cardinal numbers. For example :



etc.

Formal Unit—A unit of measure officially adopted by a government as a basis for measurement. For example, inch, pound, minute.

Graph—A representation of a number relationship in pictorial or figural form. For example :

Pictograph uses a pictorial symbol to represent a unit quantity.
Line graph uses the length along a number line to represent quantity.
Bar graph uses the length of a bar to represent quantity.
Histogram uses area to represent quantity.

Graticule—A system of spaced parallel lines dividing a region of a plane into similar shapes of equal area.

Hexagon—A polygon with six sides. A *regular* hexagon has sides of equal measure and interior angles of equal measure.

Histogram—(See **Graph**).

Horizontal—Parallel to the plane of the earth's surface.

Indicial Notation— (i) A base raised to a power

(ii) An alternative form of notation in which indices are used ; e.g.
$3544 = 3 \times 10^3 + 5 \times 10^2 + 4 \times 10 + 4$.

Intersecting (Lines)—Lines which have one point common to each. Two lines in a plane are either parallel or they intersect.

Inverse— (i) of a number (with respect to multiplication) is 1 divided by that number. Hence the inverse (or reciprocal) of 3 is $(1 \div 3)$ or $\frac{1}{3}$.

(ii) when considering the interrelationship of operations, the inverse of an operation is that operation which, when performed on the result of a given operation, annuls the first operation. For example, $28 - 13 + 13 = 28$.

Irregular Shapes—Shapes (polygons in two dimensions) which do not have sides of equal measure or interior angles of equal measure.

Isosceles (Triangle)—A triangle with two sides of equal measure.

Magnitude—A quantity consisting of a measure and the chosen unit that is assigned to an object or event to indicate its size.

Map Wheel—A wheel used to measure distances indicated on a map, especially useful, for instance, to measure lengths of roads or of rivers.

Median—The middle measurement when a set of measurements are placed in order of size. If there is no actual middle case (when the number of cases is even) then the median is the average of the two "middle" cases.

Metre—The basic unit of linear measure of the metric system (approximately 39·37 in.).

Litre—One cubic decimetre (1,000 cubic centimetres). The litre is approximately 2·1 pints. A millilitre is one thousandth part of a litre—one cubic centimetre.

Mode—The most frequently occurring number in a distribution of scores.

Obtuse (Angle)—An angle greater in measure than a right angle and less in measure than two right angles.

Octagon—A polygon having eight sides.

Odometer (Hodometer)—An instrument for measuring the distance travelled by a wheeled vehicle.

Parallelogram—A quadrilateral with both pairs of opposite sides parallel.

Pentagon—A polygon having five sides.

Perimeter—The length measure of a closed curve, e.g. the perimeter of a circle, the perimeter of a polygon. (The sum of the measures of length of the sides).

Plane (Surface)—A plane surface is one such that a straight line joining any two points in it lies entirely in the surface.

Prism—A three-dimensional object with two faces called the bases which are parallel and congruent, and with the remaining faces parallelograms formed by joining corresponding vertices of the bases.

Quadrilateral—A polygon with four sides. A simple closed curve consisting of four vertices and four line segments or intervals.

Radius—An interval (line segment) with one end point as the centre of the circle and the other end point as any point on the circle.

Random (Events)—Chance events, without bias, erratic, haphazard.

Rate—A comparison between two quantities which may be different types, for example, 40 miles per gallon, or 35 miles per hour.

Rational Number— (i) A rational number is one which can be expressed in the form $\frac{m}{n}$ where m and n are integers and n is not zero.

(ii) A number which is represented by a fraction.

Rectangle—A parallelogram with one angle a right angle (and therefore all four angles right angles.) Alternatively, it may be defined as a quadrilateral with all four angles right angles.

Rectilinear (Shapes)—Shapes bounded by "straight" lines, e.g. triangle, octagon.

Reduction—The act of changing to a different form by collecting terms, making substitutions, renaming, etc.

Region (Plane)—That part of a plane enclosed by the bounding shape ; e.g. a triangular region is that part of the plane enclosed by the triangle.

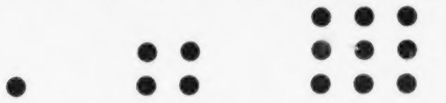
Remainder—When the operation of division is restricted to the whole numbers, a remainder may arise as the outcome of the operation of division ; for example, when 23 is divided by 4 the quotient is 5 and the remainder is 3, since $(4 \times 5) + 3 = 23$.

Scalene (Triangle)—A triangle that has no two sides of equal measure.

Sector (of a Circle)—A portion of a circular region bounded by two radii of the circle and one of the arcs which they intercept.

Square—(*a*) A *rectangle* with two adjacent sides of equal measure, *or* a parallelogram with one angle a right angle and two adjacent sides of equal measure, *or* a quadrilateral with sides of equal measure and angles of equal measure.

(*b*) *Square number*—an example of figurate number in which square arrays of dots illustrate numbers, e.g.



etc.

(*c*) *Square prism*—a prism which has square bases. (A cube also has square lateral faces. A cube is a special case of the square prism.)

(*d*) *Square pyramid*—a polyhedron with a square base and the remaining four faces triangular and meeting at a common vertex.

Statistics—Numerical facts systematically collected and calculations based on these.

Structured (Aids)—Concrete materials constructed to illustrate mathematical relationships.

Tessellate—To completely cover a surface with identical shapes without the occurrence of overlapping.

Tetrahedron—A triangular pyramid ; a polyhedron with four faces, each of which is a triangle.

Triangular Numbers—An example of figurate number such that the dots form equilateral triangular shapes.

Triangular Prism—A prism that has triangular bases and sides which form parallelograms. It thus has two triangular faces and three faces that are parallelograms (commonly rectangles).

Trundle Wheel—An odometer consisting of a wheel and a device which assists in the counting of revolutions. The circumference is commonly one yard. Thus each revolution corresponds to one yard.

Unit—(*a*) One.

(*b*) Unit of measure—an object or entity that has been chosen as a basis for comparison for the measurement of a similar object or entity.

Vertex—The point that is the intersection of three or more faces of a solid in three-dimensional space ; also the common point or origin of the two arms of an angle.

Volume—The size or the extent of the enclosed region of a closed figure is its volume. It is the numerical measure in terms of a specified unit that can be given to a solid region. It is essential that both the number and the unit used be given when a volume is stated.

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